2.13 Multipath and Diffraction Conceptual Model Specification

Multipath is a propagation path interference phenomenon that usually occurs during target track at low elevation angles. Although multipath introduces error in range and elevation target track, the most significant effect is the enhancement or cancellation of the direct path target signal energy (from radar to target) by the reflected path signal energy. The impact on target detection ranges from complete cancellation of the target return signal, resulting in no detection at a particular target range, to nearly doubling of the free space detection range. The largest effects on signal strength result from ground path reflection geometry shown in figure 2.13-1.

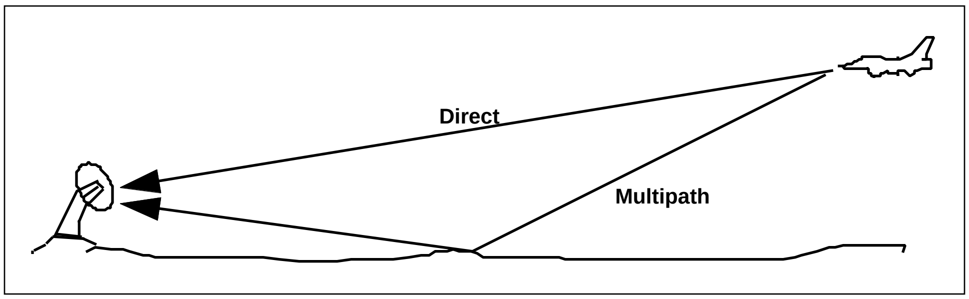

Figure 2.13-1 Ground Reflection Multipath

Multipath effects generally arise from two types of scattering processes, specular and diffuse. Specular scattering is largely due to smooth surface reflection and can be accounted for as a coherent signal return from a geometric point source. Diffuse scattering results from rough surfaces and is characterized by noncoherent reflection over a somewhat broad surface area.

The amplitude and phase of returned multipath signals are determined by the radar range equation, path geometry and scattering surface properties. The radar range equation accounts for signal strength at the target and scattering points. Path geometry dictates range distances, phase shift due to path length differences and directional transmitter and receiver gain coefficients. Scattering surface properties include surface location and orientation and roughness factors. The amount of energy reflected from a surface element is characterized by a reflection coefficient which is a function of surface roughness, wave polarization, wave length, vegetation, and the area of the illuminated surface.

In the far electromagnetic field, radio frequency energy travels as a plane wave. The wave spreads around small objects whose size is comparable to the wavelength and bends around larger obstructions. This phenomenon is called diffraction. The wave propagates in a given direction because the radiation from all points in each wave front reinforces in that direction. If there is an obstruction, the wave front is broken or distorted, reducing the reinforcement. This results in a propagation loss. Diffraction effects are generally categorized as spherical earth diffraction and knife edge diffraction. Spherical earth diffraction occurs when the edges of the wave front are intercepted by the earth. Knife edge diffraction occurs when the wave front is broken by an irregular earth surface projection.

Typically, multipath is the predominant propagation effect at distances from the radar to the radar horizon. Beyond the radar horizon, diffraction becomes the predominant effect upon propagation. Depending upon the terrain, propagation losses or gains may be due to multipath only, knife edge diffraction only, spherical earth diffraction only, or a combination of any two or all three.

The intent of the multipath and diffraction functional element in ALARM is to generate realistic signal returns that would result from propagation effects due to specular multipath, knife edge diffraction, and spherical earth diffraction. The functional element will account for physical characteristics of the reflecting terrain.

2.13.1 Functional Element Design Requirements

This section contains the design requirements necessary to fully implement the Multipath and Diffraction processing simulation.

1. The Multipath and Diffraction functional element will simulate the effects of propagation losses or gains due to specular multipath, knife edge diffraction, and spherical earth diffraction.

2. The Multipath and Diffraction functional element will allow for user-selectable roughness and dielectric characteristics of the terrain as well as terrain heights from user-selectable DMA data.

3. The Multipath and Diffraction functional element will use the methodology and algorithms of the Spherical Earth/Knife Edge (SEKE) model developed by the MIT Lincoln Laboratory to calculate the propagation loss or gain due to specular multipath, knife edge diffraction, and/or spherical earth diffraction.

2.13.2 Functional Element Design Approach

This section discusses the design elements that implement design requirements outlined in the previous section. A design element is an algorithm that represents a specific component of the FE design.

The ALARM multipath and diffraction functional element calculates a pattern propagation factor based only on terrain points that lie in the vertical plane defined by the radar, the target, and the center of the earth. Propagation effects due to points outside this plane are considered negligible and are not considered. This factor is computed as a weighted combination of individual factors due to specular multipath, knife edge diffraction, and spherical earth diffraction. This weighted combination allows target and terrain geometry to determine which of the three effects contribute to the overall pattern propagation factor, and to what extent. ALARM assumes these effects are the same in both directions; therefore, only one-way propagation need be examined.

Decision Logic and Terrain Design Elements

The first design elements concern decisions based on geometric relationships between the radar site, the target, and the ground terrain. Based on these relationships, weighting factors for the three effects are defined. These design elements support design requirements 1 and 3 of Section 2.13.2. In support of design requirement 2, the terrain profile used in the computations of all effects is derived from the user-selected DMA terrain data base.

Design Element 13-1: Propagation Effects Decision Logic

Signal propagation over the spherical earth is traditionally divided into three regions. These regions are known as (1) the interference region where the target is well above the horizon and multipath is predominant, (2) the diffraction region, where the target is sufficiently far below the horizon (i.e., masked) that diffraction is the principal effect, and (3) the intermediate region along the line-of-sight to the horizon where both multipath and diffraction occur.

In the ALARM simulation of these effects, there are two main decisions made, as described in [A.1-13]. The first decision is whether to use a line-of-sight model (multipath) and/or a diffraction model. If the program decides to use a diffraction model, a second decision must be made: to use knife edge diffraction or spherical earth diffraction. Figure 2.13-2 contains a diagram of the geometry and the decision logic.

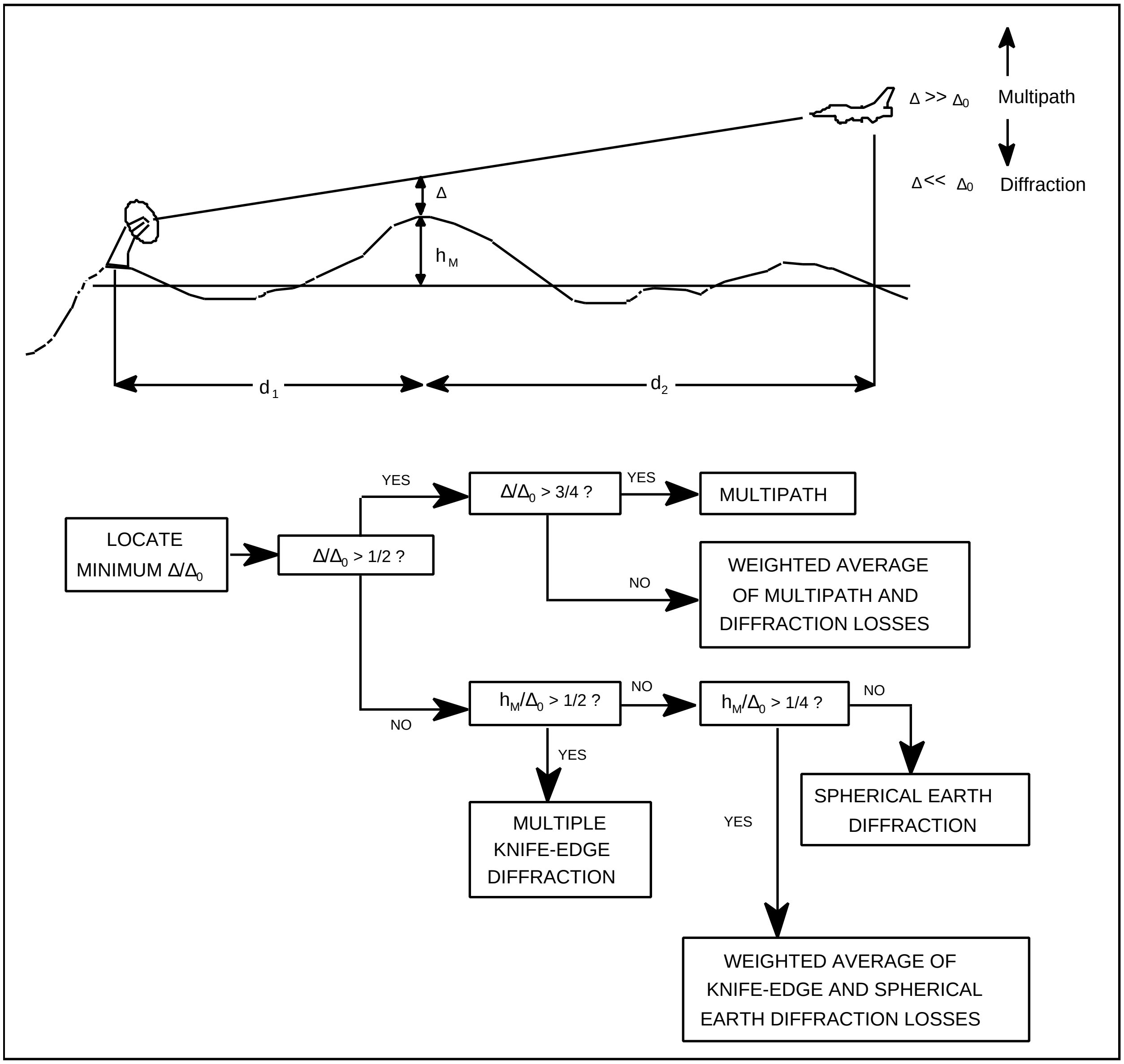

Figure 2.13-2 Propagation Effects Decision Logic

In general, if propagation effects are to be considered, the propagation effects factor F is calculated as follows [A.1-13, equations 2 and 3]:

$$F = \begin{cases} F_M & \text{if } \frac{\Delta}{\Delta_0} \geq \frac{3}{4} \\ \alpha_{M,D} F_M + (1 - \alpha_{M,D}) F_D & \text{if } \frac{1}{2} \leq \frac{\Delta}{\Delta_0} < \frac{3}{4} \\ F_D \end{cases} \tag{2.13-1}$$

where F_M = multipath factor
F_D = diffraction factor
$\alpha_{M,D}$ = $4(\Delta/\Delta_0) - 2$
Δ/Δ_0 is a clearance ratio defined in Design Element 13-2

If $\Delta/\Delta_0 \leq 3/4$, then equations 4 and 5 of [A.1-13] indicate that the (combined) diffraction effects factor F_D generally is calculated as follows:

$$F_D = \begin{cases} F_K & \text{if } \frac{h_M}{\Delta_0} \geq \frac{1}{2} \\ \alpha_{K,S} F_K + (1 - \alpha_{K,S}) F_S & \text{if } \frac{1}{4} \leq \frac{h_M}{\Delta_0} < \frac{1}{2} \\ F_S & \text{if } \frac{h_M}{\Delta_0} < \frac{1}{4} \end{cases} \tag{2.13-2}$$

where F_K = knife edge diffraction factor
F_S = spherical earth diffraction factor
$\alpha_{K,S}$ = $4(h_M/\Delta_0) - 1$
h_M/Δ_0 is defined in Design Element 13-3

The first decision is based on the clearance of the direct ray between the radar and the target. Clearances are defined for all terrain points between the radar and the target.

Design Element 13-2: Clearance Ratios

Based on equation 1 of [A.1-13], the Fresnel clearance ($\Delta_{0,i}$) at each such terrain point i is defined as follows:

$$\Delta_{0,i} = \left(\frac{\lambda d_{1,i} d_{2,i}}{d_{1,i} + d_{2,i}}\right)^{1/2} \tag{2.13-3}$$

where λ = radar wavelength (meters)
$d_{1,i}$ = ground range from radar to terrain point i (meters)
$d_{2,i}$ = ground range from terrain point i to target (meters)

Next, the clearance (Δ_i) at each terrain point i is defined as the vertical distance from the terrain to the line of sight between the radar and the target [A.1-13, page 4]:

$$\Delta_i = h_a + d_{1,i} \tan \varepsilon_t - z_i \tag{2.13-4}$$

where h_a = antenna height (meters above mean sea level)
$d_{1,i}$ = ground range from radar to terrain point i (meters)
ε_t = elevation angle from antenna to target
z_i = terrain height at i (meters above mean sea level)

Finally, D is defined as min $\{\Delta_i/\Delta_{0,i}\}$ over all terrain points i; and Δ and Δ_0 are defined to be the clearances associated with the point yielding this minimum; i.e.,

$$\frac{\Delta}{\Delta_0} = D = min_i \left\{ \frac{\Delta_i}{\Delta_{0,i}} \right\} \tag{2.13-5}$$

Design Element 13-3: Diffraction Effects Ratio

As discussed in [A.1-13, pages 8-9], the determination of the diffraction effect is based on the ratio of h_M to Δ_0, where Δ_0 is the Fresnel clearance defined in equation (2.13-5)) and h_M is defined as follows. Obtain the best straight line fit (see Design Element 13-25) to the terrain profile $\{(x_i, h_i)\}$, where x_i is the ground range from the radar to the i^{th} point (called $d_{1,i}$ above) and h_i is the height of this terrain point (called z_i above). Let the equation of this line be given as follows:

$$h = a_0 + a_1 x \tag{2.13-6}$$

Let i_0 be the index of the point which yielded Δ/Δ_0 above. Then h_M is defined to be the terrain height at i_0 when measured with respect to the line given in equation (2.13-6)); i.e.,

$$h_M = h_{i_0} - (a_0 + a_1 x_{i_0}) \tag{2.13-7}$$

Design Element 13-4: Factors for Special Cases

Exceptions to the definitions in equations (2.13-1)) and (2.13-2)) are necessary if the spherical earth diffraction series does not converge (see Design Element 13-19). The discussion in [A.1-13], leads to the following algorithm for determining F in this case:

If $\frac{\Delta}{\Delta_0} < \frac{3}{4}$ and $\frac{1}{4} \le \frac{h_M}{\Delta_0} < \frac{1}{2}$ (i.e., a combination of spherical earth and knife edge diffraction were to be used), but the spherical earth diffraction series does not converge, then

$F_D = F_K$ and F is defined as in equation (2.13-1)

(2.13-8)

If $\frac{\Delta}{\Delta_0} < \frac{3}{4}$ and $\frac{h_M}{\Delta_0} < \frac{1}{4}$ (i.e., no knife edge diffraction was to be used), but the spherical earth diffraction series does not converge,then the propagation effects factor F is defined as follows:

$$F = \begin{cases} F_K & \text{if the target is masked} \\ F_M & \text{if the target is unmasked} \end{cases}$$

Design Element 13-5: Cases of No Propagation Effects

If the user chooses not to consider propagation effects (user input IPROP = 0) or if the ground range between the radar and the target is less than or equal to the terrain grid size, then

$$F = \begin{cases} 0 & \text{if target is masked} \\ \sqrt{G_T} & \text{otherwise} \end{cases} \tag{2.13-9}$$

where G_T = antenna gain of the radar transmitter

Design Element 13-6: Application of Propagation Effects

The propagation effects factor F above is calculated on the basis of electric field strength for one-way propagation. It must be squared to obtain the corresponding factor for signal power, and then squared again to account for two-way propagation. Thus, F^4 is the factor to be multiplied times the target signal received at the antenna.

This SEKE logic is also used to calculate the multipath/diffraction effects of on-board noise and deception jammers and stand-off noise jammers. For on-board jamming, the propagation factor calculated for the target signal return is also used as the factor for the jammer. For stand-off jammers, the terrain profile from the radar to the jammer is used to calculate the propagation effect, using the equations and algorithms described above. In both jammer cases, the propagation is one-way rather than two-way, so F^2 is multiplied times the jammer signal, rather than F^4.

Design Element 13-7: Terrain Profile

The terrain profile used in determining the effects of multipath and diffraction consists of terrain points in the plane determined by the radar, the target, and the center of the earth. The points chosen are not necessarily in the input terrain data base, but are defined to be approximately the same distance apart as the data base points. Thus, the number of points, N_P, in the terrain profile is defined by the following equation:

$$N_P = \ldots N_I - 1 \tag{2.13-10}$$

where

N_I	=	$[[G_R / \Delta G + 0.5]]$
G_R	=	ground range from radar to target (m)
ΔG	=	distance between two adjacent points in terrain data base at same longitude (m)
[[ ]]	=	denotes the greatest integer function

Note that ΔG does not depend on target location, so it is calculated during initialization.

$$\Delta G = \left(\frac{3}{3600}\right)\left(\frac{\pi}{180}\right)R_0 \qquad (2.13\text{-}11)$$

where	R_0	=	radius of earth (6,371,007 m)
	3/3600	=	number of degrees of latitude between DMA terrain points
	$\pi/180$	=	conversion factor for degrees to radians

Design Element 13-8: Terrain Visibility

The determination of the visibility of the points in the terrain profile is a complex geometric process that begins with computing the coordinates of each point in the profile. The first step is to determine the latitude of the i^{th} profile point.

$$\sin\varphi_i = \cos\varphi_s \sin\beta_i \cos\alpha_t + \sin\varphi_s \cos\beta_i \qquad (2.13\text{-}12)$$

where	φ_i	=	latitude of i^{th} profile point
	φ_s	=	latitude of radar site
	α_t	=	azimuth of target (and hence all profile points) with respect to site
	β_i	=	distance from site to i^{th} terrain point

Note that $ß_i$ does not depend on target position; it is calculated during run initialization.

$$\beta_i = \frac{i\Delta G}{K_R R_0} \qquad (2.13\text{-}13)$$

where	K_R	=	refractivity factor (nominally 4/3)
	ΔG and R_0 are defined in equation (2.13-11))		

The calculations of φ_s and α_t are not part of this FE. The next step is to calculate the longitude.

$$\lambda_i = \lambda_s + \tan^{-1}\left(\frac{\sin\alpha_t \sin\beta_i}{\cos\varphi_s \cos\beta_i - \sin\varphi_s \sin\beta_i \cos\alpha_t}\right) \qquad (2.13\text{-}14)$$

where	λ_i	=	longitude of i^{th} profile point
	λ_s	=	longitude of site
	other variables defined above		

The calculated latitude and longitude are compared to the positions of the input terrain data base points to determine the height above mean sea level (MSL) of the profile point. This algorithm is also used for other purposes, it is not examined as part of this FE.

Next, a two-dimensional (x,z) coordinate system is defined with its origin at MSL with radar site latitude and longitude. The coordinates of the ith terrain point are calculated as follows:

$$\begin{aligned} x_i &= (R_e + h_i)\sin\beta_i' \\ z_i &= (R_e + h_i)\cos\beta_i' - R_e \end{aligned} \qquad (2.13\text{-}15)$$

where	R_e	=	R_0/K_R (effective radius of earth)
	h_i	=	height above MSL of ith terrain point (m)
	β_i'	=	β_i/K_R
	β_i	=	true ground distance from radar to target at MSL (m)
	K_R	=	refractivity factor (nominally 4/3)

Note that R_e and β_i' are used instead of R_0 and β_i to account for refractivity when considering terrain geometry propagation effects.

The elevation angle from the radar antenna to the ith profile point is calculated

$$\varepsilon_i = \tan^{-1}\left(\frac{z_i - h_a}{x_i}\right) \qquad (2.13\text{-}16)$$

where	ε_i	=	elevation angle from radar antenna to ith profile point
	h_a	=	height of antenna above MSL
	(x_i,z_i)	=	coordinates of ith profile point

Finally, the determination of the visibility of the ith profile point is determined by the following algorithm:

Point i is visible to the radar if and only if

$$\varepsilon_j < \varepsilon_i \text{ for } j = 1, 2, \ldots, i-1 \qquad (2.13\text{-}17)$$

where	ε_k	=	elevation angle from radar antenna to kth profile point

Multipath Design Elements

The multipath effects determined by ALARM are specular reflection contributions from the terrain points in the plane defined by the radar, the target, and the center of the earth. The program looks at each point in the terrain between the radar and the target and excludes those points which are masked from either the antenna or the target. The remaining points are examined to find specular reflection points; i.e., those points for which the incident ray from the radar is reflected in a ray that intersects the target (see figure 2.13-1).

In this document, the first Fresnel zone associated with any specular point is defined to be the set of points for which the difference between the specular path length and the bounce path length from radar to target through the point is less than half a wavelength (see Design Element 13-12 below). This is different from the classical optics definition of a Fresnel zone in that it uses the specular path length rather than the direct path length; however, this terminology agrees with [A.1-16, page 11]. The contribution to the multipath effect factor F_M from each specular point is calculated as a function of the width of the first Fresnel zone around that point.

The following algorithms consist of top-level equations for the multipath factor and equations for subfactors of this value. These elements support design requirements 1, 2, and 3 of Section 2.13.1.

Design Element 13-9: Multipath Effects Factor

The multipath factor F_M includes contributions from the direct path and all specular paths.

$$F_M = \begin{cases} max(TEMP, 10^{-10}) & \text{if } \sum_{i=1}^{N} w_i > 0 \\ \sqrt{G_D} & \text{otherwise} \end{cases}$$

where (2.13-18)

$$TEMP = \left| \sqrt{G_D} + \frac{\sum_{i=1}^{N} \left(\rho_i \rho_s w_i \sqrt{G_B} \exp\left(\frac{2\pi j \delta_i}{\lambda} \right) \right)}{\sum_{i=1}^{N} w_i} \right|$$

where	G_D	=	antenna gain along direct path from radar to target
	N	=	number of specular points considered
	ρ_i	=	reflection coefficient of smooth plane tangent to bounce point
	ρ_s	=	correction factor for terrain roughness
	w_i	=	width of Fresnel zone associated with i^{th} specular point
	G_B	=	antenna gain along path from radar to bounce point
	δ_i	=	difference in length of bounce path and direct path (meters)
	λ	=	wavelength (meters)
	j	=	square root of -1

Note that if there are no specular points or if no specular point has a nonzero Fresnel zone width, then the multipath factor collapses to account only for the gain along the direct path. Equation (2.13-18)) was derived empirically and does not mimic SEKE logic precisely; SEKE uses Fresnel zone width differently. However, equation (2.13-18)) reduces to equation 6.3 in [A.1-11] for a smooth plane (N = 1 and ρ_s = 0).

The reflection coefficient ρ_i of a smooth plane depends on grazing angle and is discussed in the following paragraph. The terrain roughness correction factor ρ_s is discussed in the paragraph after that. Thus, the following two design elements support design requirement 2 of Section 2.13.1.

Design Element 13-10: Reflection Coefficient of a Smooth Plane

According to [A.1-25, page 260, equations 6.69 - 6.71], the reflection coefficient ρ_i of a smooth plane is given by

$$\rho_i = \begin{cases} \dfrac{y^2 \sin\gamma - \sqrt{y^2 - \cos^2\gamma}}{y^2 \sin\gamma + \sqrt{y^2 - \cos^2\gamma}} & \text{for vertical polarization} \\[2ex] \dfrac{\sin\gamma - \sqrt{y^2 - \cos^2\gamma}}{\sin\gamma + \sqrt{y^2 - \cos^2\gamma}} & \text{for horizontal polarization} \end{cases} \qquad (2.13\text{-}19)$$

where γ = grazing angle of reflected ray

and

$$y^2 = \begin{cases} \varepsilon_1 - j(60\lambda\sigma) & \text{over land} \\[2ex] \dfrac{\varepsilon_1 - 4.9}{1 + x^2} + 4.9 + j\left[\dfrac{x(\varepsilon_1 - 4.9)}{1 + x^2} + \dfrac{2\sigma(9\ \ 10^{10})}{f}\right] & \text{over sea} \end{cases} \qquad (2.13\text{-}20)$$

where
ε_1 = relative dielectric constant of surface
λ = radar wavelength (meters)
σ = terrain conductivity (mho/m)
x = $2\pi f\tau$
f = radar frequency (hertz)
τ = relaxation constant
j = square root of -1

This equation assumes that the magnetic permeability of the terrain equals that of free space.

Design Element 13-11: Terrain Roughness Factor

Over land, the terrain roughness correction factor (ρ_S in equation (2.13-18))) is constant over the entire scenario, and hence is a user input. Over sea, this factor varies with windspeed and grazing angle. The factor is given by

$$(a) \quad \rho_s = \exp\left(\rho_w \sin^2 \gamma\right)$$

where

$$(b) \quad \rho_w = -2\left(\frac{2\pi\sigma_w}{\lambda}\right)^2 \qquad (2.13\text{-}21)$$

and

$$(c) \quad \sigma_w = \frac{1}{\sqrt{2\pi}}\left(\frac{v}{8.67}\right)^{\frac{5}{2}}$$

where v = wind speed (m/sec)
γ = grazing angle of reflected ray
λ = radar wavelength (meters)

These equations are based on [A.1-25, pages 260-267].

Design Element 13-12: Width of First Fresnel Zone

As described in [A.1-16, pages 11-13], the first Fresnel zone around a specular point is assumed to lie in the plane tangent to the terrain point. ALARM uses only that part of the Fresnel zone that is on the projection of the line connecting the antenna and the target. This line is shown as the x-axis in figure 2.13-3.

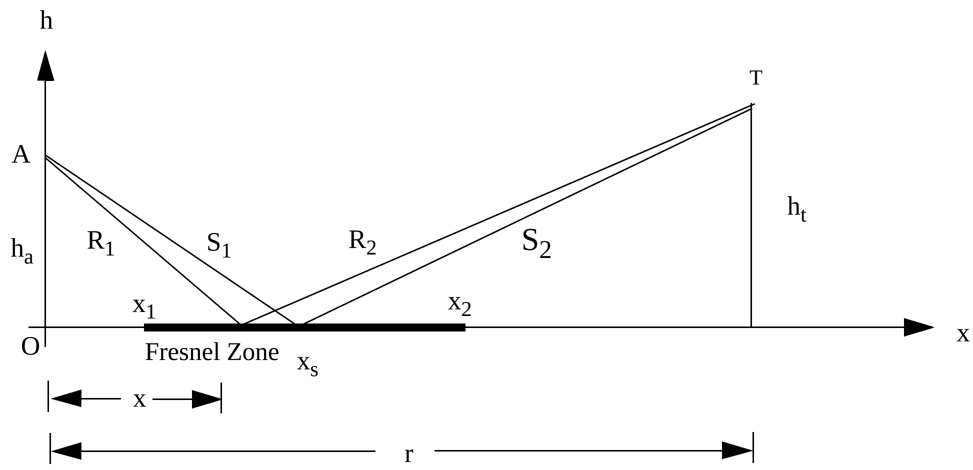

Figure 2.13-3 Fresnel Zone Geometry

Using the notation of figure 2.13-3, the first Fresnel zone is defined [A.1-16 page 11] to consist of all points (x,0) such that

$$R_1 + R_2 - R_S \leq \frac{\lambda}{2} \qquad (2.13\text{-}22)$$

where

where	R_1	=	$[x^2 + h_a^2]^{\int}$ = range from antenna to (x,0)
	R_2	=	$[(r - x)^2 + h_t^2]^{\int}$ = range from (x,0) to target
	R_S	=	$S_1 + S_2$ = specular path length
	S_1	=	$[x_s^2 + h_a^2]^{\int}$
	S_2	=	$[(r - x_s)^2 + h_t^2]^{\int}$
	λ	=	wavelength
	h_a	=	antenna height from tangent plane
	h_t	=	target height from tangent plane
	r	=	"ground" range from antenna to target (along tangent plane)
	x	=	"ground" range from antenna to (x,0)
	x_s	=	"ground" range to specular point

Thus, the width w of the first Fresnel zone is equal to $x_2 - x_1$ where x_1 and x_2 are the solutions of the bounding equation

$$R_1 + R_2 - R_S = \frac{\lambda}{2} \qquad (2.13\text{-}23)$$

Note again that this is not the classical optics definition of Fresnel zone. It can be shown algebraically that w can be calculated as follows:

$$w = x_2 - x_1 = \frac{[b^2 + 4a(-c)]^{\frac{1}{2}}}{a}$$

where

$$a = \left(R_S + \frac{\lambda}{2}\right)^2 - r^2 \qquad (2.13\text{-}24)$$

$$b = r(h_t^{\,2} - h_a^{\,2} - a)$$

$$-c = \frac{1}{4}(h_a^{\,2} + h_t^{\,2} - a)^2 - h_a^{\,2}(r^2 + h_t^{\,2})$$

Design Element 13-13: Specular Reflection Points

Every point i in the terrain profile which is visible to both the radar and the target is examined to determine whether or not it is considered a specular reflection point; i.e., a point at which the reflected ray intersects the target. It is unlikely that any terrain profile point itself is an actual specular point, since terrain profile points occur only once every 90 meters. However, for simplicity, if there are any specular points at all in the terrain, they will be assumed to occur at one of the nearest points in the terrain profile.

For each point i in the terrain profile, the terrain on either side of point i is considered; figure 2.13-4 shows a possible configuration. In this case, the ray R_i is the reflection at i relative to the slope of the preceding terrain (from point i-1 to point i), and ray L_i is the reflection at i relative to the slope of the following terrain (from point i to point i+1). Thus, R_i is the reflected ray at i with respect to the terrain interval for which i is the <u>right</u> end point, and L_i is the reflected ray at i with respect to the terrain for which i is the <u>left</u> end point. Point i is taken to be a specular reflection point if L_i lies on the opposite side of the target from R_i; i.e., if

$$(\tan\varphi_i - m_i)(\tan\Psi_i - m_i) < 0 \qquad (2.13\text{-}25)$$

where	$\tan \Psi_i$	=	slope of ray L_i
	$\tan \varphi_i'$	=	slope of ray R_i
	m_i	=	slope of line from point i to target

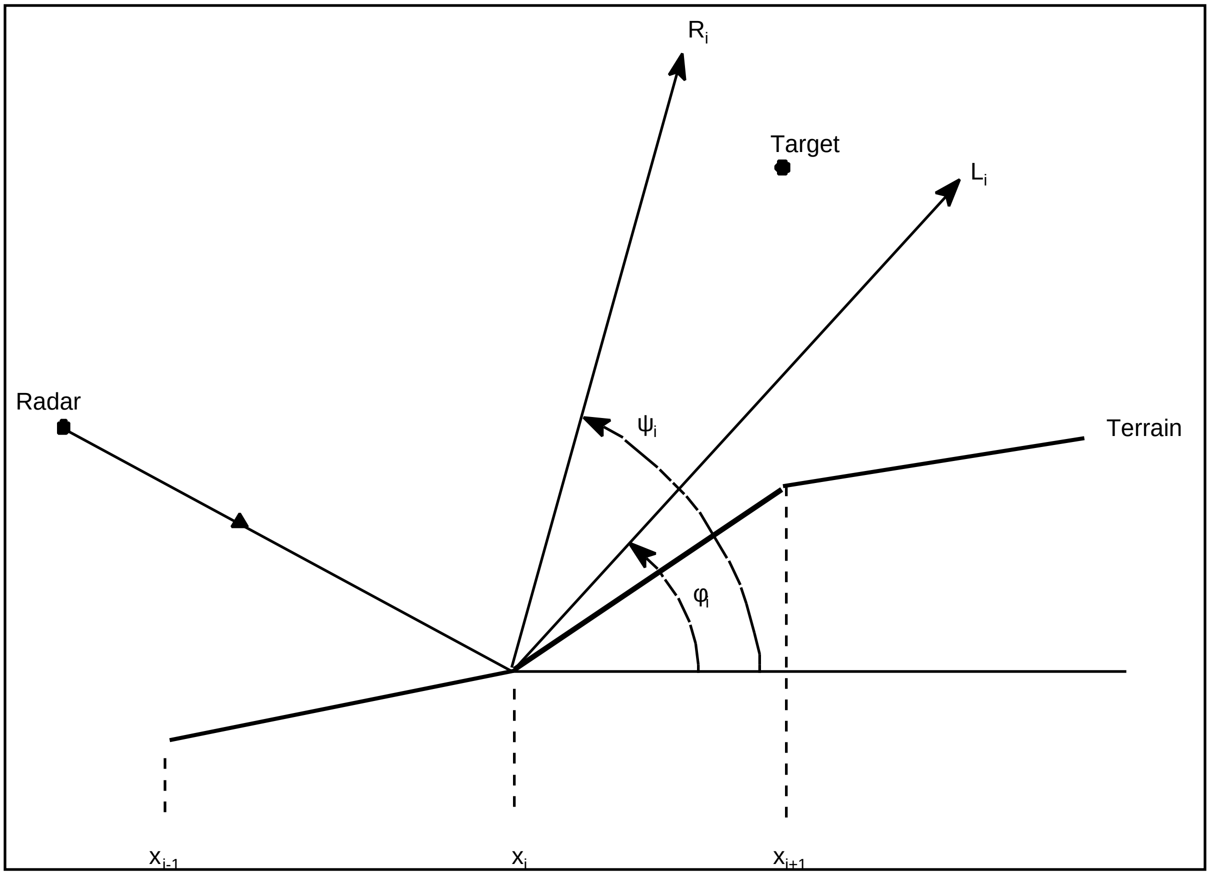

Figure 2.13-4 Specular Point Geometry I

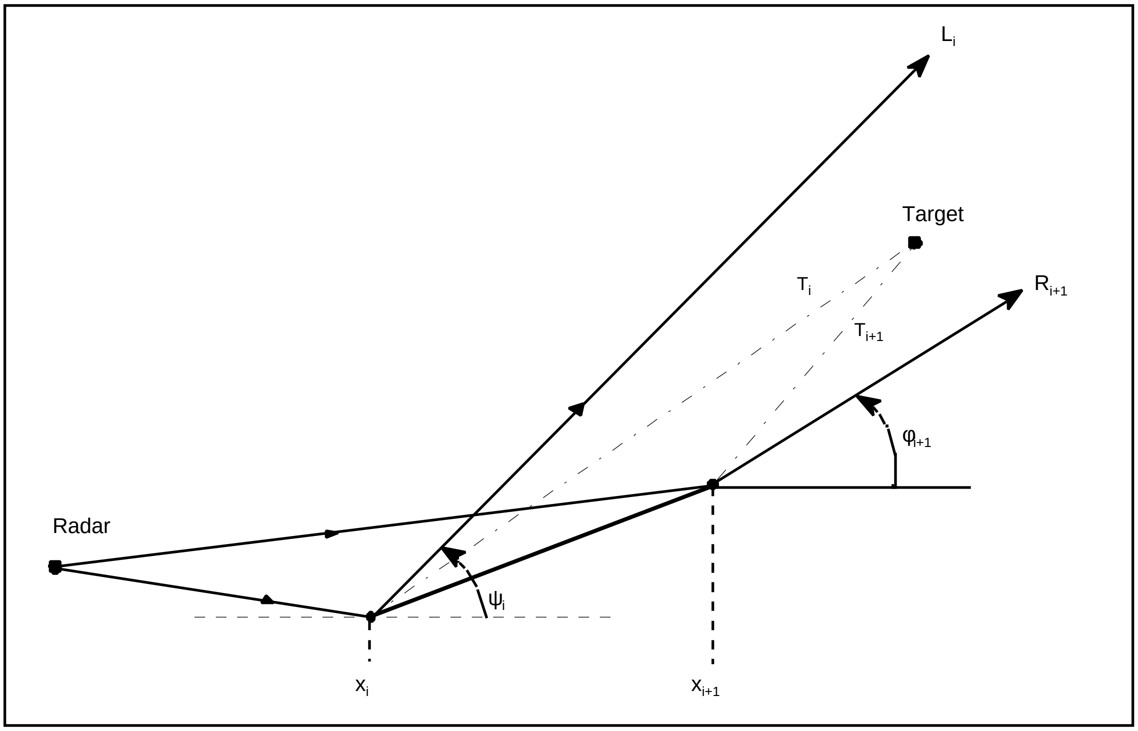

Figure 2.13-5 Specular Point Geometry II

If i is not a specular reflection point based on equation (2.13-25)), then the geometry of the reflected rays at point i and point i+1 relative to the terrain between them is determined. Figure 2.13-5 shows a possible geometric configuration. Ray L_i is the reflection from point i relative to the terrain patch between points i and i+1, while ray R_{i+1} is the reflection from point i+1 relative to the same terrain patch from i to i+1.

If point i itself were a specular reflection point, the target would lie on ray L_i. In general, there is a specular reflection point between point i and point i+1 if ray L_i lies above T_i (the ray from point i to the target) and ray R_{i+1} lies below T_{i+1} (the ray from point i+1 to the target); i.e., if

$$\tan\Psi_i < m_i \textit{ and } \tan\Phi_{i+1} > m_{i+1} \tag{2.13-26}$$

where	$\tan\Psi_i$	=	slope of ray L_i
	$\tan\Phi_{i+1}$	=	slope of ray R_{i+1}
	m_i	=	slope of T_i
	m_{i+1}	=	slope of T_{i+1}

If this is the case, then for ease of calculation, point i itself is taken to be the specular reflection point.

In either of the two cases above, a plane through point i is found so that the incident grazing angle equals the reflected grazing angle at i with respect to that plane (see figure 2.13-6). The grazing angle γ_i is calculated with respect to this plane.

In figure 2.13-6, note that ε_i, the elevation of the terrain point (x_i, z_i) with respect to the radar, is negative, while α_i, the elevation angle from the point (x_i, z_i) to the target, is positive. The angles $90 + \varepsilon_i$ and $90 - \alpha_i$ are determined by right triangles. Since the sum of these two angles plus $2\gamma_i$ must be 180, the grazing angle γ_i is given by

$$\gamma_i = \frac{1}{2}(\alpha_i - \varepsilon_i) \qquad (2.13\text{-}27)$$

where ε_i = elevation angle from radar to point i
$\tan \alpha_i$ = slope of line from point i to the target

If point i satisfies one of the two conditions above, the path lengths are further examined to check whether there is an overlap of the direct and reflected pulse returns; i.e., point i is kept as a specular point if

$$R_S - R_D < \tau \qquad (2.13\text{-}28)$$

where R_S = specular path length at point i (meters)
R_D = direct path length from radar to target (meters)
τ = pulse width (meters)

ALARM allows only one specular point per visible area, the one associated with the largest first Fresnel zone in that area. So the final step in determining the specular points to be used is to examine all specular points in a visible area and record only that one with the largest first Fresnel zone. After this is done for all visible areas, equation (2.13-18)) is used to calculate the multipath factor.

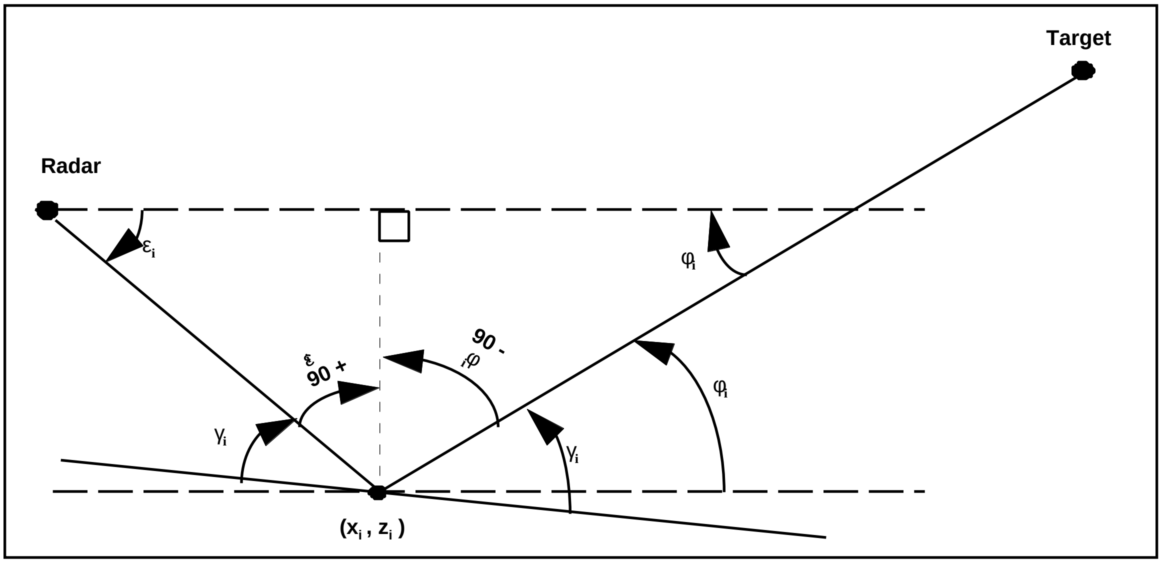

Figure 2.13-6 Specular Point Reflection Angle

Knife Edge Diffraction Design Elements

The knife edge diffraction factor is computed using Deygout's Method for up to three diffraction points; the main knife edge diffraction point M, a diffraction point L between the main diffraction point and the radar, and a diffraction point R between the main knife edge diffraction point and the target. This methodology is based on [A.1-13, pages 17-19] and [A.1-11, pages 27-30 and 34-35]. Again, only points in the plane defined by the radar, the target and the center of the earth are considered.

The following algorithms are top-level and detailed methods for computing the factor for knife edge diffraction. These support design requirements 1, 2, and 3 of Section 2.13.1.

Design Element 13-14: Locate Knife Edges

The knife edge diffraction points are certain local maximum terrain points between the radar and the target. A terrain point is defined to be a local maximum if and only if it has an elevation angle (relative to the antenna) greater than the elevation angles of the next two terrain points on both sides; i.e.

$$\text{Terrain point } (x_i, y_i) \text{ is a local maximum if and only if} \quad \varepsilon_i > \varepsilon_{i+k} \quad k = \pm 1, \pm 2 \tag{2.13-29}$$

where ε_j = elevation angle of terrain point j with respect to the radar antenna

The main knife edge diffraction point M is defined to be the local maximum terrain point which has a minimum value of Δ/Δ_0, where Δ and Δ_0 are defined as in equations (2.13-3) and (2.13-4). For greater fidelity, one or two secondary knife edge diffraction points (L and R) may be used, one on each side of M. Each of these must also be a local maximum with the minimum Δ/Δ_0 on their respective side of M.

$$M = (x_M, z_M) \text{ is the main knife edge diffraction point if and only if M is a local maximum, and } \frac{\Delta_M}{\Delta_{0,M}} \leq \frac{\Delta_i}{\Delta_{0,i}} \text{ for all other local maxima} (x_i, z_i) \tag{2.13-30}$$

$$L = (x_L, z_L) \text{ is the left knife edge diffraction point if and only if L is a local maximum lying between the radar site and point M, and } \frac{\Delta_L}{\Delta_{0,L}} \leq \frac{\Delta_i}{\Delta_{0,i}} \text{ for all other local maxima} (x_i, z_i) \text{ between the radar site and point M.} \tag{2.13-31}$$

$$R = (x_R, z_R) \text{ is the right knife edge diffraction point if and only if R is a local maximum lying betwen point M and the target, and } \frac{\Delta_R}{\Delta_{0,R}} \leq \frac{\Delta_i}{\Delta_{0,i}} \text{ for all other local maxima} (x_i, z_i) \text{ between point M and the target.} \tag{2.13-32}$$

Design Element 12-15 below describes how Δ and Δ_0 are defined for R and L. It has been shown [A.1-13, page 18] that good results are obtained by requiring the knife edge diffraction points to be separated from each other by about 900 meters. This corresponds to a separation of ten terrain points in the DMA terrain data base tables used in ALARM.

Design Element 13-15: Clearances for Knife Edge Diffraction

For the main knife edge diffraction point M = (x_M, z_M), the clearances Δ_i and $\Delta_{i,0}$ are defined as in equations (2.13-3)) and (2.13-4)). However, for the secondary diffraction points (L and R) Δ_i and $\Delta_{i,0}$ are defined in terms of the main knife edge diffraction point (see [A.1-12, pages 34-35]). Figures 2.13-7 and 2.13-8 show possible geometries for the secondary knife edge diffraction point between the radar and the main diffraction point M. This is called the left diffraction point L = (x_L, z_L).

If the target is unmasked, then Δ_L is defined by equation (2.13-4) as shown in figure 2.13-7. However, if the target is masked as in figure 2.13-8, then Δ_L is defined to be the clearance between terrain point L and the ray from the antenna to the main diffraction point M.

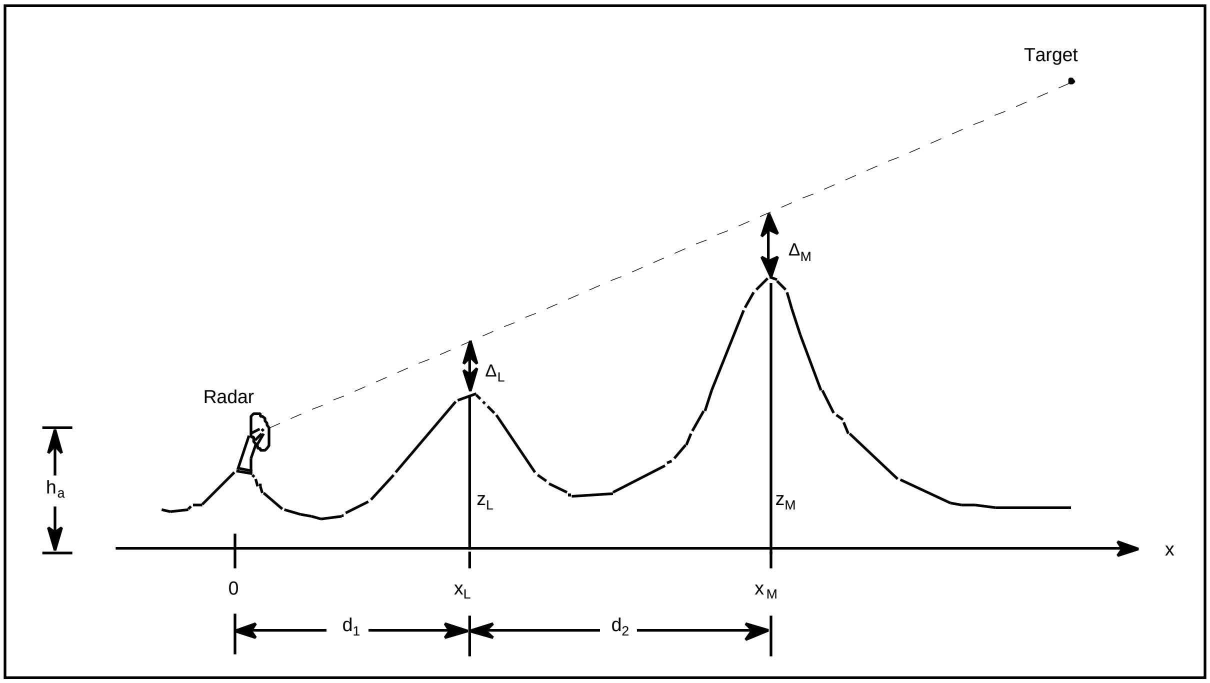

Figure 2.13-7 Knife Edge Diffraction Geometry (Target Unmasked)

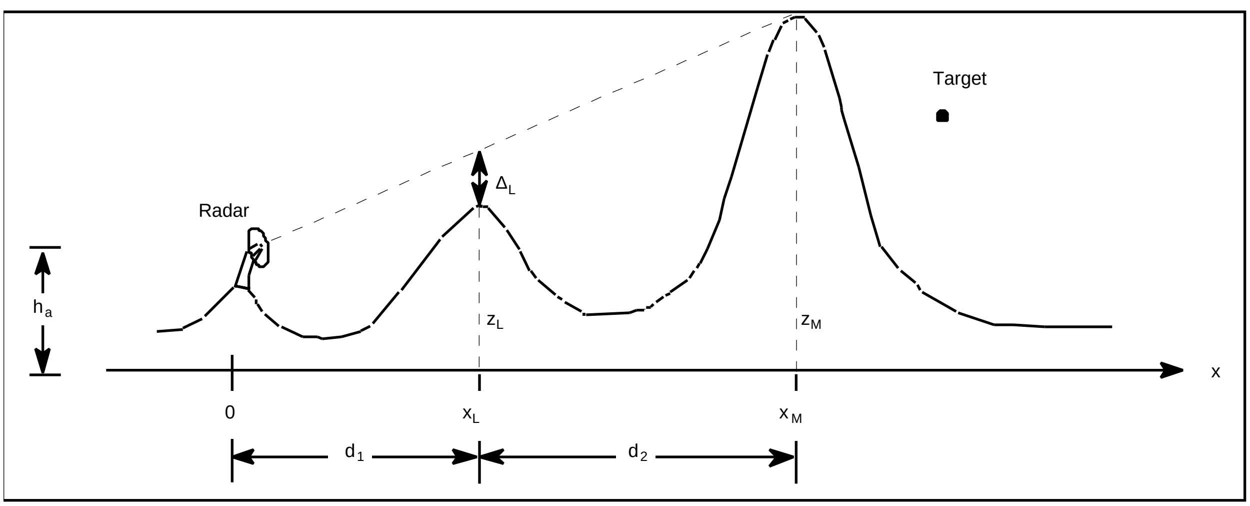

Figure 2.13-8 Knife Edge Diffraction Geometry (Target Masked)

Thus the clearance Δ_L is defined as follows:

$$\Delta_L = \begin{cases} \Delta_{i_L} & \text{equation (2.13-4)} \quad \text{if target unmasked} \\ h_a + d_1 \tan \varepsilon_M - z_L & \text{if target masked} \end{cases} \tag{2.13-33}$$

where h_a = height of antenna
d_1 = x-distance from radar to point L
ε_M = elevation angle of main diffraction point
z_L = height of terrain point L

Furthermore, for the left diffraction point

$$\Delta_{0,L} = \sqrt{\frac{\lambda d_1 d_2}{d_1 + d_2}} \tag{2.13-34}$$

where λ = wavelength
d_1 = x-distance from radar to point L
d_2 = x-distance from point L to point M (main diffraction point)

Equations (2.13-33)) and (2.13-34)) also apply to the secondary diffraction point between M and the target (point R), by replacing "L" by "R" in the equations and by redefining d_1 and d_2 as follows:

$$\begin{aligned} d_1 &= \text{x-distance from M to R} \\ d_2 &= \text{x-distance from R to target} \end{aligned} \tag{2.13-35}$$

Design Element 13-16: Individual Knife Edge Diffraction

The propagation effects factor F_i contributed by each knife edge diffraction point (i = L, M, R) is calculated as follows:

$$F_i = \begin{cases} 1 & \text{if } \dfrac{\Delta_i}{\Delta_{0,i}} \geq 100 \\ \dfrac{\sqrt{\left[\frac{1}{2} + C(w_i)\right]^2 + \left[\frac{1}{2} + S(w_i)\right]^2}}{\sqrt{2}} & \text{otherwise} \end{cases} \tag{2.13-36}$$

where $C(w_i)$ = Fresnel cosine integral defined in Section 2.3.3-11

$S(w_i)$ = Fresnel sine integral defined in Section 2.3.3-11

w_i = width of first Fresnel zone at point i

Δ_i and $\Delta_{0,i}$ are clearances defined in Section 2.3.3.10

The first part of equation (2.13-36)) is based on the assumption that knife edge diffraction is negligible at points with very large clearances. The limiting value of 100 meters is suggested in [A.1-13, page 23].

The second part of equation (2.13-36)) is based on [A.1-11, equation 2.1], where the knife edge diffraction propagation effects factor F_i associated with terrain point i can be expressed as

$$F_i = \left|\frac{E_i}{E_0}\right| \tag{2.13-37}$$

where E_0 = free space electrical field

E_i = electrical field including knife edge diffraction at point i

In addition, for one way propagation, equation 5.1 of [A.1-11] gives E_i as

$$E_i = \frac{e^{-j\frac{\pi}{4}} e^{jkr}}{\sqrt{2}r} \left(\left[\frac{1}{2} + C(w)\right] + j\left[\frac{1}{2} + S(w)\right] \right) \quad (2.13\text{-}38)$$

where r = range from antenna to target
k = $2\pi/\lambda$
j = square root of -1
w is defined as

$$w = \sqrt{2}\left(\frac{\Delta_i}{\sqrt{\frac{\lambda d_T d_R}{(d_T + d_R)}}} \right) \quad (2.13\text{-}39)$$

where d_T = distance from transmitter to terrain point
d_R = distance from receiver to terrain point
λ = wavelength
Δ_i is defined above for i = L, M, R

The second part of equation (2.13-36)) can be obtained as follows. The free space field E_0 in equation (2.13-37)) can be found from equations (2.13-38)) and (2.13-39)) by letting Δ_i approach ∞. Equation (2.13-39)) then yields that w approaches ∞. Furthermore, equation 4.15 in [A.1-17] yields

$$C(\infty) = S(\infty) = \frac{1}{2} \quad (2.13\text{-}40)$$

Substituting into equation (2.13-38)) gives

$$E_0 = \frac{e^{-j\frac{\pi}{4}} e^{jkr}}{\sqrt{2}r}(1 + j) \quad (2.13\text{-}41)$$

Finally, substituting (2.13-41)) and (2.13-38)) into (2.13-37)) yields the second part of (2.13-36)).

Design Element 13-17: Combined Knife Edge Diffraction Factor

After equation (2.13-36)) has been used to find the factors F_i, i = L, M, R, the total knife edge diffraction factor F_K is computed as

$$F_K = \sqrt{\mathcal{G}} F_L \quad F_M \quad F_R \tag{2.13-42}$$

where $\mathcal{G}$ is the antenna gain in the direction of the target, if it is unmasked. If the target is masked, $\mathcal{G}$ is the gain in the direction of the highest mask point; i.e., the terrain point with the minimum value of Δ/Δ_0. This method for determining gain is based on [A.1-15, pages 10-12]. (ALARM is designed so that antenna gain calculations are performed within this functional element.)

The factors for the three points are constant for any signal input, so they are multiplied to denote the successive changes to signal strength from point to point.

Design Element 13-18: Fresnel Sine and Cosine Functions

The Fresnel sine and cosine functions are defined as follows:

$$\begin{aligned} S(x) &= \int_0^x \sin\left(\frac{\pi t^2}{2}\right) dt \\ C(x) &= \int_0^x \cos\left(\frac{\pi t^2}{2}\right) dt \end{aligned} \tag{2.13-43}$$

[A.1-11, appendix A, pp 56-57] contains a FORTRAN program that numerically evaluates C(x) as follows:

$$C(x) = \int_0^x \cos\left(\frac{\pi t^2}{2}\right) dt \approx c + k\left[\cos \mathrm{z} \sum_{i=1}^{12} a_i z^{12-i} + \sin \mathrm{z} \sum_{i=1}^{12} b_i z^{12-i}\right] \tag{2.13-44}$$

where

$$\left.\begin{array}{l} z = \frac{1}{4}\pi x^2 \\ c = 0 \\ k = \sqrt{\frac{\pi}{8}}\ |x| \\ a_i = a_{i1} \quad i = 1, \quad 12 \\ b_i = b_{i1} \quad i = 1, \quad 12 \end{array}\right\} if \quad \frac{\pi}{2}x^2 \le 4 \qquad (2.13\text{-}45)$$

and

$$\left.\begin{array}{l} z = 4\frac{\pi}{2}x^2 \\ c = .5 \\ k = \frac{\pi}{2}\frac{1}{|x|} \\ a_i = a_{i2} \quad i = 1,12 \\ b_i = b_{i2} \quad i = 1,12 \end{array}\right\} if \quad \frac{\pi}{2}x^2 > 4 \qquad (2.13\text{-}46)$$

Where $a_{i\,j}$ and $b_{i\,j}$ are constants defined in the reference.

S(x) is calculated using identical equations with different values of a_i and b_i, i = 1,...12.

ALARM calculates the sums in C(x) and S(x) using the following recursive formulations: For j = 1,...,12

$$\begin{array}{l} C_j = A_j + zC_{j-1} \\ S_j = B_j + zS_{j-1} \end{array} \qquad (2.13\text{-}47)$$

where	A_j	=	a_{12-j}
	B_j	=	b_{12-j}
and	C_o	=	k cos z
	S_o	=	k sin z

This formulation is computationally efficient because there are fewer products to calculate. It also protects against underflow, since there is no direct calculation of terms on the order of z^{12}.

Spherical Earth Diffraction Design Elements

The spherical earth diffraction factor is calculated using Fock's series, as described in [A.1-13, A.1-14, A.1-15]. The following design elements for spherical earth diffraction effects support design requirements 1 and 3 of Section 2.13.1.

Design Element 13-19: Spherical Earth Diffraction Effects Factor

Fock has shown [A.1-14] that the propagation factor, F_S, is a function of normalized antenna height, y, normalized target height, z, and normalized range, x. Combining this result with the antenna pattern factor described in [A.1-15, pages 10-11], leads to the following form for F_s:

$$F_S = \sqrt{\mathcal{G}}F(x, y, z) \tag{2.13-48}$$

where $\mathcal{G}$ is defined as in equation (2.13-42)) above, and F(x,y,z) is Fock's series, which is defined as follows in [A.1-14, equation 29]:

$$F(x, y, z) = 2\sqrt{\pi x}\sum_{n=1}^{\infty}\overline{f_n(y)}\,\overline{f_n(z)}\exp\left[-\psi - \zeta + \frac{1}{2}(\sqrt{3} + j)a_n x\right] \tag{2.13-49}$$

where x, y, and z are the normalized values referred to above, and the bar over f_n in equation (2.13-49)) does **not** denote the complex conjugate, but denotes a function defined in terms of $\overline{\mathrm{Ai}}(z)$, a variant (not the complex conjugate) of the Airy function.

$$\overline{f_n(u)} = \frac{\overline{Ai}\left(a_n + \exp\left(\frac{\pi j}{3}\right)u\right)}{\exp\left(\frac{\pi j}{3}\right)Ai'(a_n)} \tag{2.13-50}$$

and

$$\begin{aligned}
\overline{Ai}(w) &= Ai(w)\exp\left(\frac{2}{3}w^{\frac{3}{2}}\right) \\
Ai &= \text{Airy Function} \\
Ai' &= \text{derivative of Airy Function} \\
a_n &= n^{th} \text{ zero of Airy Function} \\
\psi &= \frac{2}{3}y^{\frac{3}{2}} \\
\zeta &= \frac{2}{3}z^{\frac{3}{2}} \\
j &= \sqrt{-1}
\end{aligned} \tag{2.13-51}$$

The Airy Function is discussed in Design Element 13-21. Normalized values of range and height are defined in Design Element 13-20.

Up to 35 terms of the infinite series in equation (2.13-49)) are computed. If two successive terms have absolute value less than 0.0005, then the series is assumed to converge to the value of the current partial sum. If the thirty-fifth term is reached without satisfying this criterion, or if the absolute value of any term is greater than 10,000, then the series is assumed to be divergent [A.1-14, pages 19-20].

Design Element 13-20: Normalized Values of Ground Range and Heights

The normalization process begins with finding a parabolic fit to the terrain profile (see [A.1-13, Section 2.2]). Specifically, a parabola

$$h = a_0 + a_1 x + a_2 x^2 \tag{2.13-52}$$

is found which is the best least-squares fit to the terrain points (x_i,h_i) between the radar and the target (see Design Element 13-26).

According to the SEKE code (listed in [A.1-15, page 43]), the effective earth radius for spherical earth diffraction is defined as follows:

$$\frac{1}{R_{eff}} = \frac{1}{R_e} - \frac{2a_2}{K} \tag{2.13-53}$$

where R_e = effective radius of earth for atmospheric refraction effects
a_2 = coefficient of x^2 in parabola (equation (2.13-51)))
K = atmospheric refraction factor included in R_e (usually 4/3)

If $R_{eff} > 0$, then the intermediate values y' and z' respectively are defined to be the heights of the antenna and the target above the parabola; i.e.,

$$\begin{aligned} y' &= h_a - a_0 \\ z' &= h_t - (a_0 + a_1 r + a_2 r^2) \end{aligned} \tag{2.13-54}$$

where h_a = antenna height above mean sea level
h_t = target height above mean sea level
r = ground range from antenna to target

If $R_{eff} \leq 0$, then the terrain profile is concave, and hence a straight line fit will be used instead of a parabolic fit [A.1-13, Section 2.2]. This line is given by

$$h = a_0 + a_1 x \tag{2.13-55}$$

In this case R_{eff} becomes R_e and the intermediate values y' and z' are defined to be the heights of antenna and target above the straight line.

$$\begin{aligned} y' &= h_a - a_0 \\ z' &= h_t - (a_0 + a_1 r) \end{aligned} \tag{2.13-56}$$

In either case, if the terrain is such that a negative effective height is obtained, it is replaced by λ/4, where λ is the wavelength in meters. ([A.1-15, page 43]); i.e.,

$$\begin{aligned} &If \quad y' < 0 \quad then \quad y' = \frac{\lambda}{4} \\ &If \quad z' < 0 \quad then \quad z' = \frac{\lambda}{4} \end{aligned} \tag{2.13-57}$$

Finally, from [A.1-14, pages 5-6], the normalized range, antenna height, and target height are found as follows:

$$x = r\left(\frac{\pi}{\lambda R_{eff}^2}\right)^{\frac{1}{3}}$$

$$y = 2y'\left(\frac{\pi^2}{\lambda^2 R_{eff}}\right)^{\frac{1}{3}} \qquad (2.13\text{-}58)$$

$$z = 2z'\left(\frac{\pi^2}{\lambda^2 R_{eff}}\right)^{\frac{1}{3}}$$

where r is the ground range from the radar to the target and y' and z' were defined in equations (2.13-47)) or (2.13-49)) and (2.13-50)).

Design Element 13-21: Airy Region Determination

The Airy Function is defined to be a complex function Ai(z) which satisfies the differential equation

$$\frac{d^2(Ai)}{dz^2} - z \quad Ai = 0 \qquad (2.13\text{-}59)$$

According to [A.1-14, page 10], Ai may be expressed as

$$Ai(z) = \frac{1}{\pi}\int_0^\infty \cos\left(\frac{1}{3}t^3 + zt\right)dt \qquad (2.13\text{-}60)$$

As noted in Design Element 13-19 above, ALARM uses $\overline{\text{Ai}}$(z), a modified form of the Airy function (not its complex conjugate), to calculate Fock's series.

$$\overline{Ai}(z) = Ai(z)\exp\left(\frac{2}{3}z^{\frac{3}{2}}\right) \qquad (2.13\text{-}61)$$

To calculate the value of $\overline{\text{Ai}}$(z), the complex plane is partitioned into three regions. Region 1 (Connection Region) consists of all complex numbers z such that

$$\begin{gathered} Re(z) < 0 \\ and \\ |\arg \; z| > \frac{2\pi}{3} \end{gathered} \tag{2.13-62}$$

Region 2 (Integral Region) consists of all complex numbers z such that

$$\begin{gathered} Re(z) < 0 \;\; and \;\; |\arg \; z| \le \frac{2\pi}{3} \;\; and \;\; |z| > 4 \\ OR \\ Re(z) \ge 0 \;\; and \;\; |z| > 2 \end{gathered} \tag{2.13-63}$$

Region 3 consists of all complex numbers z such that

$$\begin{gathered} Re(z) < 0 \;\; and \;\; |\arg \; z| \le \frac{2\pi}{3} \;\; and \;\; |z| \le 4 \\ OR \\ Re(z) \ge 0 \;\; and \;\; |z| \le 2 \end{gathered} \tag{2.13-64}$$

These regions are shown in figure 2.13-9. This methodology is based on [A.1-14, pages 11-12].

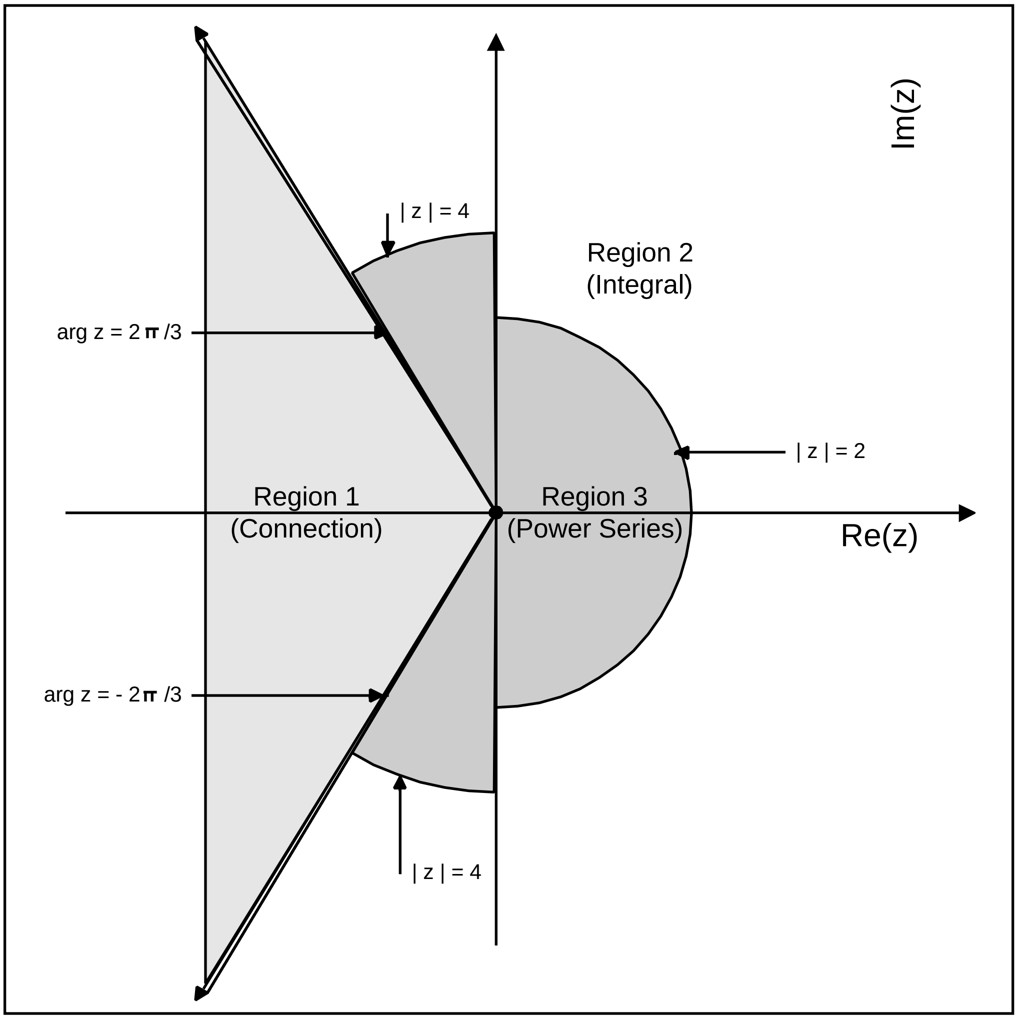

Figure 2.13-9   Airy Function Regions on the Complex Plane

Design Element 13-22: Airy Function in Connection Region

If z is in the Connection Region, so that |arg z| > 2π/3, then $\overline{Ai}$(z) is calculated in terms of $\overline{Ai}$(w) and $\overline{Ai}$(v) where w and v lie outside of region 1. Specifically, from equation 30 of [A.1-14],

$$\overline{Ai}(z) = e^{\frac{\pi}{3}j}\,\overline{Ai}\left(ze^{\left(-\frac{2\pi}{3}\right)j}\right)e^{-\frac{4}{3}u^{\frac{3}{2}}} + e^{-\frac{\pi}{3}j}\,\overline{Ai}\left(ze^{\frac{2\pi}{3}j}\right) \qquad (2.13\text{-}65)$$

where j = square root of -1

u = x + iy

x = min (Re(z), -64)

y = Im(z) if Im(z) $\geq$ -64

Im(z)(mod 2π) if Im(z) < -64

Note that u is used instead of z to prevent underflow in the exponential computation.

Design Element 13-23: Airy Function in Integral Region

If z is in the Integral Region of figure 2.13-9, then $\overline{\mathrm{Ai}}(z)$ is calculated using a corrected version of equation 28 of [A.1-14] (the summand in equation 28 was changed to match the code on p 40 of [A.1-14]):

$$\overline{Ai}(z, N) = e^{\zeta} Ai(z, N) = e^{\zeta}\left\{\frac{1}{2}\pi^{-\frac{1}{2}} z^{-\frac{1}{4}} e^{-\zeta} \sum_{i=1}^{N} \frac{w_i}{1 + \frac{x_i}{\zeta}}\right\} \qquad (2.13\text{-}66)$$

$$where \quad \zeta = \frac{2}{3} z^{\frac{3}{2}}$$

and where x_i and w_i are the abscissas and weights given in Table III on page 18 of [A.1-14]. ALARM uses N = 10 as recommended on page 17 of [A.1-14].

Design Element 13-24: Airy Function in Power Series Region

If z is in the Power Series Region of figure 2.13-9, then $\overline{\mathrm{Ai}}(z)$ is calculated using the Taylor series expansion given in equations 12 - 17 of [A.1-14].

$$\begin{aligned}
\overline{Ai}(z) &= Ai(z)e^{\zeta} \\
Ai(z) &= \alpha \, h(z) - \beta \, g(z) \\
\\
where \quad \alpha &= 0.35502805 \\
\beta &= 0.2488194 \\
h(z) &= 1 + \frac{1}{3!}z^3 + \frac{1 \cdot 4}{6!}z^6 + \cdots + \frac{1 \cdot 4 \cdots (3k-2)}{(3k)!}z^{3k} + \cdots \\
g(z) &= z + \frac{2}{4!}z^4 + \frac{2 \cdot 5}{7!}z^7 + \cdots + \frac{2 \cdot 5 \cdots (3k-1)}{(3k+1)!}z^{3k+1} + \cdots
\end{aligned} \qquad (2.13\text{-}67)$$

This expression can be rewritten as

$$\overline{Ai}(z) = (\alpha - \beta z) + \sum_{i=1}^{\infty} [\alpha h_i(z) - \beta g_i(z)] \qquad (2.13\text{-}68)$$

where α, ß, h_i, and g_i are the terms expressed in equation (2.13-67)).

ALARM calculates these series recursively using the following formulas:

$$
\begin{aligned}
h_o(z) &= 1 \\
h_{i+1}(z) &= \frac{h_i(z)z^3}{(3i+2)(3i+3)} \qquad for \quad i = 0, 1 \\
g_o(z) &= 1 \\
g_{i+1}(z) &= \frac{g_i(z)z^3}{(3i+3)(3i+4)} \qquad for \quad i = 0, 1
\end{aligned}
\tag{2.13-69}
$$

The number of terms to be used in computing this infinite sum, M, is given by rounding the value 7 + 4 |z| to the nearest integer. This value for M was determined empirically to ensure that the last term computed is smaller than 10^{-10}; i.e.,

$$|\alpha h_M(z) - \beta g_M(z)| < 10^{-10} \tag{2.13-70}$$

Design Element 13-25: Linear Fit

ALARM uses a standard least-squares algorithm to find the equation of a line that fits a set of points. This is used in the calculation of the diffraction effects ratio (Design Element 13-3).

The coefficients a_0 and a_1 of the line $y = a_0 + a_1x$ that best fits the set of points $\{(x_1,y_1), (x_2,y_2),. . . ,(x_n,y_n)\}$ are given by the following equations.

For N > 2

$$a_1 = \frac{\sum_{i=1}^{N} x_i y_i - \frac{1}{N}\sum_{i=1}^{N} x_i \sum_{i=1}^{N} y_i}{\sum_{i=1}^{N} x_i^2 - \frac{1}{N}\left(\sum_{i=1}^{N} x_i\right)^2} \tag{2.13-71}$$

and

3636

$$a_0 = \frac{1}{N}\sum_{i=1}^{N} y_i - \frac{a_1}{N}\sum_{i=1}^{N} x_i$$

If N = 2, then the line is exactly and uniquely determined:

$$a_1 = \frac{y_2 - y_1}{x_2 - x_1} \quad and \quad a_0 = y_1 - a_1 x_1 \qquad (2.13\text{-}72)$$

If N = 1, then any line through the point could be used; ALARM selects the horizontal line.

$$a_1 = 0 \quad and \quad a_0 = y_1 \qquad (2.13\text{-}73)$$

Design Element 13-26: Parabolic Fit

ALARM uses a standard least-squares algorithm to find the equation of a parabola that best fits a set of points. This is used in the calculation of normalized ground range and heights in spherical earth diffraction computations (Design Element 13-20).

The coefficients a_0, a_1, and a_2 of the parabola $a_0 + a_1x + a_2x^2$ that best fits the set of points $\{(x_1,y_1), (x_2,y_2),. . . ,(x_N,y_N)\}$ are given by the following equations.

For N > 2,

$$a_2 = \frac{A_{11}B_2 - A_{12}B_1}{A_{11}A_{22} - A_{12}A_{12}}$$

and

$$a_1 = \frac{B_1 - A_{12}a_2}{A_{11}}$$

and

$$a_0 = \frac{\sum_{i=1}^{N} y_i - a_1 \sum_{i=1}^{N} x_i - a_2 \sum_{i=1}^{N} x_i^2}{N}$$

where

$$A_{11} = \sum_{i=1}^{N} x_i^2 - \frac{1}{N}\left(\sum_{i=1}^{N} x_i\right)^2 \qquad (2.13\text{-}74)$$

$$A_{12} = \sum_{i=1}^{N} x_i^3 - \frac{1}{N}\sum_{i=1}^{N} x_i \sum_{i=1}^{N} x_i^2$$

$$A_{22} = \sum_{i=1}^{N} x_i^4 - \frac{1}{N}\left(\sum_{i=1}^{N} x_i^2\right)^2$$

$$B_1 = \sum_{i=1}^{N} x_i y_i - \frac{1}{N}\sum_{i=1}^{N} x_i \sum_{i=1}^{N} y_i$$

$$B_2 = \sum_{i=1}^{N} x_i^2 y_i - \frac{1}{N}\sum_{i=1}^{N} x_i^2 \sum_{i=1}^{N} y_i$$

For N = 2, two points determine a line, so

$$a_2 = 0$$

$$a_1 = \frac{y_2 - y_1}{x_2 - x_1} \qquad (2.13\text{-}75)$$

$$a_0 = \frac{x_2 y_1 - x_1 y_2}{x_2 - x_1}$$

For N = 1, ALARM selects the horizontal line through the point.

$$\begin{aligned} a_2 &= 0 \\ a_1 &= 0 \\ a_0 &= y_1 \end{aligned} \tag{2.13-76}$$

2.13.3 Functional Element Software Design

This section describes the software design necessary to implement the functional element requirements and the design approach outlined above. Section 2.13.3 is organized as follows: the first part describes the subroutine hierarchy and gives descriptions of the relevant subroutines; the next three parts contain logical flow charts and describe all important operations represented by each block in the charts; the last part contains a description of all input and output data for the functional element as a whole and for each subroutine which implements multipath and diffraction.

Multipath and Diffraction Subroutine Design

The FORTRAN call tree that is implemented for the Multipath and Diffraction Functional Element in the ALARM 3.0 source code is shown in figure 2.13-13. The diagram depicts the entire model's structure for this functional element, from ALARM (the main program) through the least significant subroutine in the Multipath and Diffraction functional element. Subroutines which directly implement the functional element appear as shaded blocks. Subroutines which use functional element results appear with bands at the ends. Each of these subroutines is briefly described in table 2.13-1.

Table 2.13-1 Subroutine Descriptions

MODULE NAME	DESCRIPTION
AIRY	Performs an Airy function evaluation
BIODMA	Updates terrain buffer from disk
CONECT	Determines value of an Airy function in "connection" region of complex plane (region 1)
DEYGOU	Finds knife edges and calculates knife edge diffraction
ELEV	Gets elevation data for specified point
FIRST	Determines minimum ratio of clearance of direct ray to Fresnel clearance at each point in terrain profile
FRESNL	Determines Fresnel sine and cosine integrals
GAUSSQ	Determines value of an Airy function in "integral representation" region of complex plane (region 2)

Table 2.13-1 Subroutine Descriptions

MODULE NAME	DESCRIPTION
GEOMTR	Calculates variables for radar/target geometry for multipath/diffraction
GETNDX	Performs binary search of vector of points to determine left hand index of an input interval
JAMMER	Calculates signal strength of target return for pulse doppler radars
KEDIFF	Determines knife edge diffraction loss
LAPROP	Determines correct combination of multipath and diffraction effects
LINFIT	Calculates a linear fit function
MLTPTH	Calculates multipath contribution to target signature
OFFBOR	Calculates off bore-sight azimuth and elevation angles
PARFIT	Calculates a parabolic fit
POWERS	Determines value of an Airy function in "power series" region of complex plane (region 3)
PROFIL	Builds a terrain profile for ground trace of ray from site to target (or jammer)
RDRERR	Checks bounds on user-defined radar parameters
RDRINP	Reads in user-defined radar parameters
RDRINT	Initializes user-defined radar parameters
RDRPRT	Prints user-defined radar parameters
RFLECT	Determines complex reflection coefficient of a plane earth
RGAIN	Determines receiver antenna gain for specified off-boresight angles
SAVRST	Preserves/restores radar to target terrain profile common variables to allow use of common multipath/ diffraction logic for stand-off jammers
SECOND	Determines ratio of highest mask to Fresnel clearance at that point
SEDIFF	Determines spherical earth diffraction loss
SEKERR	Confirms limits on user-input data for multipath/diffraction
SEKINP	Reads user-input data for multipath/diffraction
SEKINT	Initializes internal variables for multipath/diffraction
SEKPRT	Prints user-input data for multipath/diffraction
TARGET	Computes sines and cosines of target latitude and of target orientation angles
TGAIN	Determines transmitter antenna gain for specified off-boresight angles
TARGPD	Calculates signal strength of target return for pulse doppler radars
VISIBLE	Sets up terrain profile, determines if each point is visible from radar
Note: Modules implementing multipath and diffraction functional element are identified in **bold** letters.	

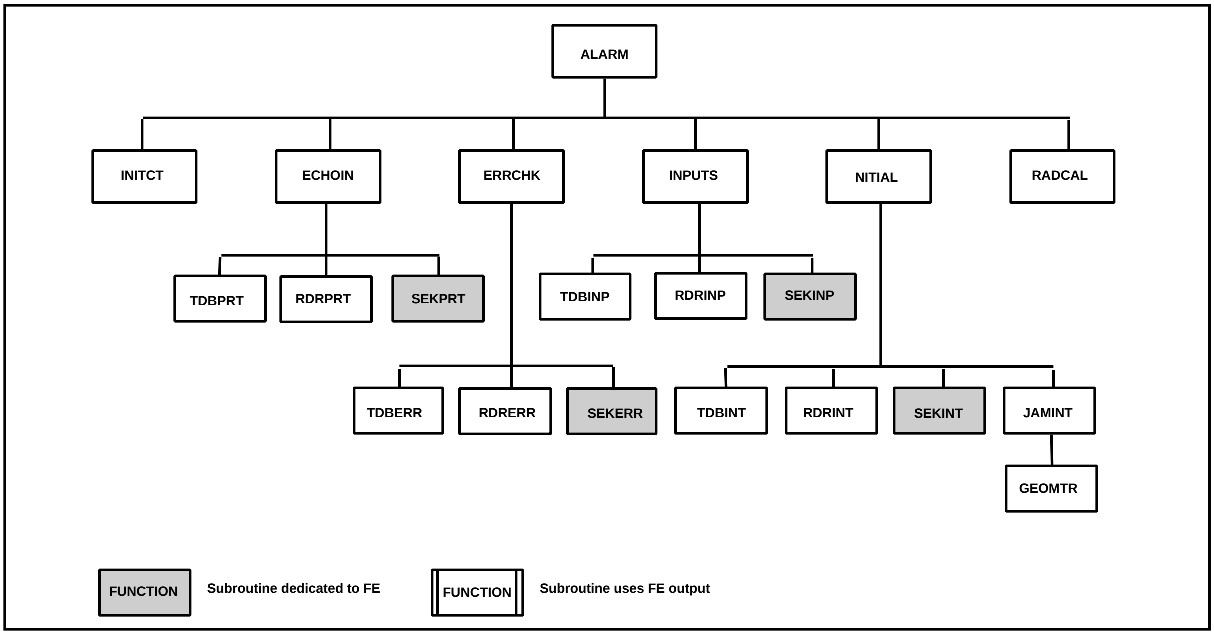

Figure 2.13-10a   Call Hierarchy for Multipath/Diffraction

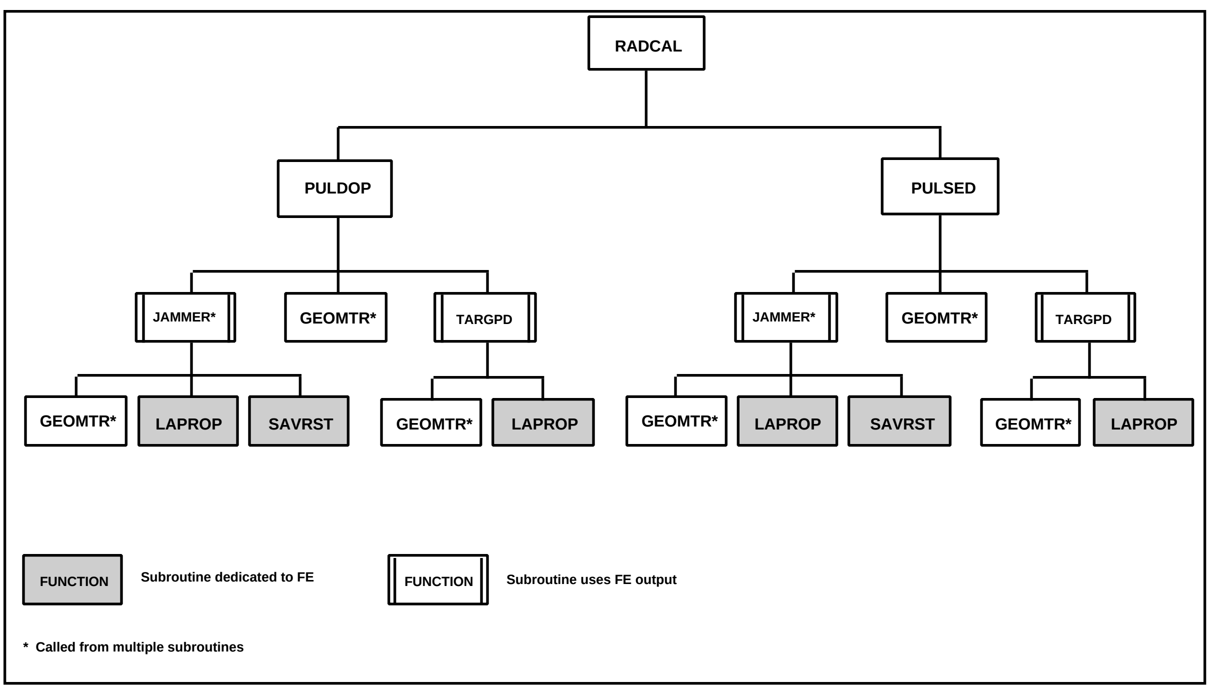

Figure 2.13-10b   Call Hierarchy for Multipath/Diffraction (continued)

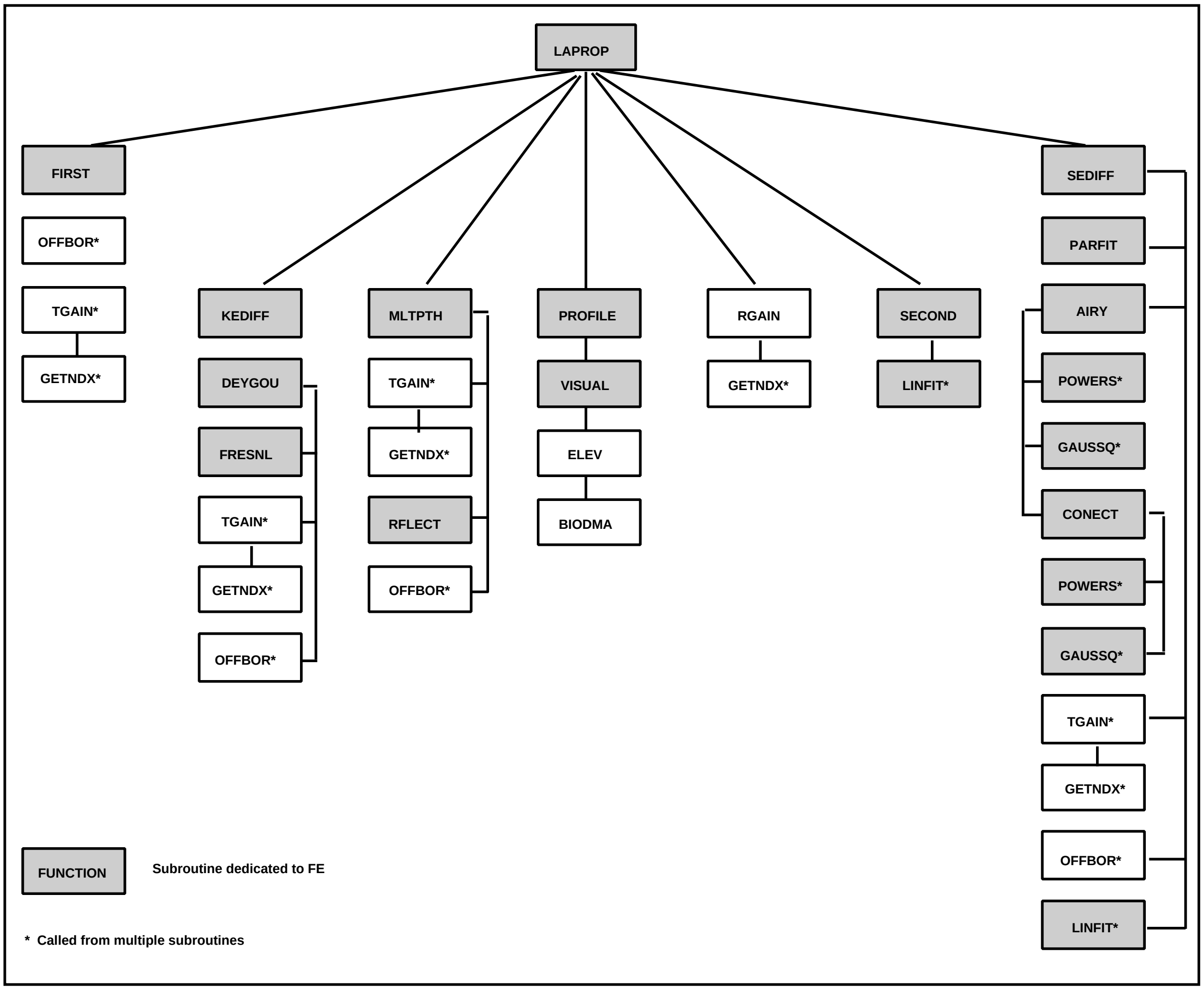

Figure 2.13-10c   Call Hierarchy for Multipath/Diffraction (continued)

Functional Flow Diagram

Figure 2.13-11 shows the top-level logical flow of the Multipath and Diffraction functional element implementation. Subroutine names appear in parentheses at the bottom of each process block. The numbered blocks are described below.

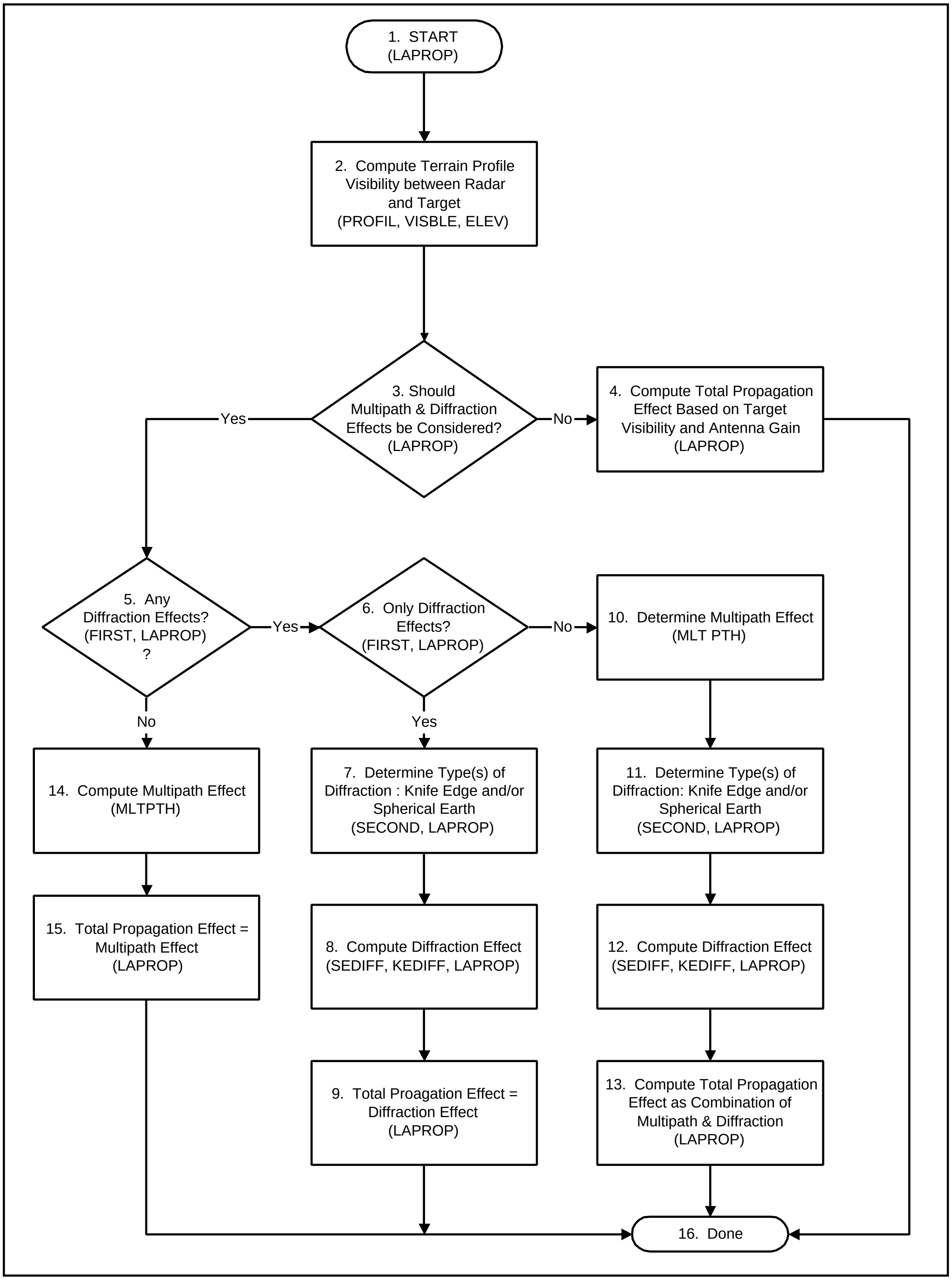

Figure 2.13-11 Multipath/Diffraction Logical Flow

Block 1: The top-level subroutine in the Multipath/Diffraction functional element is LAPROP. Figure 2.13-11 shows the general logic of LAPROP and its subroutines in the calculation of the propagation effects factor F.

Block 2: LAPROP calls PROFIL to determine the number N of evenly-spaced terrain points on the ground line between the radar and the target. This is based on the range between radar and target and on the grid size of the DMA terrain data. If N = 0, then the propagation effects factor F is calculated according to equation (2.13-9)).

PROFIL calls VISBLE to calculate the coordinates (latitude and longitude) of the N points in the terrain profile from the known coordinates of the radar and the target.

VISBLE calls ELEV to access the DMA terrain data and determine the elevations of the N points in the terrain profile. VISBLE then calculates (x_i, z_i), i = 1, N, where x_i is the ground range from the radar to the i^{th} point and z_i is the height of the terrain at the i^{th} point. The elevation angle ε_i of each terrain point relative to the antenna is also calculated, and the ε_i are all compared to determine which points are visible to the radar antenna. PROFIL then determines if the target is masked or unmasked.

Blocks 3 and 4: If the user has selected that no propagation effects be considered (i.e., if LPPROP = FALSE), then equation (2.13-9)) is used to calculate F. If LPPROP = TRUE, then the following blocks are performed.

Block 5: This block marks the start of the implementation of the algorithm displayed in figure 2.13-2 and expressed in equations (2.13-1)) through (2.13-8). LAPROP calls FIRST to compute Δ/Δ_0 as described in equations (2.13-1)) through (2.13-3)). FIRST also calculates and saves the i_0 which produces this minimum.

Then LAPROP compares Δ/Δ_0 to 0.75 (see equation (2.13-1))) to determine whether or not to consider any effects due to diffraction; i.e., whether or not to calculate F_D. If diffraction effects are to be considered, LAPROP calls SECOND to compute h_M as described in equation (2.13-7)), recalculate Δ_0, and then compute h_M/Δ_0.

Block 6: If some propagation effects are due to diffraction, LAPROP compares Δ/Δ_0 to 0.5 (see equation (2.13-1))) to determine if there are also effects due to multipath.

Block 7: If no multipath, only diffraction effects are applicable, comparison of h_M/Δ_0 to 0.25 and 0.5 (see equation (2.13-2))) is then used to determine whether to calculate a knife edge diffraction effects factor (F_K), a spherical earth diffraction effects factor (F_S), or both.

Blocks 8 and 9: If required, KEDIFF (figure 2.13-12) is called to compute F_K, and SEDIFF (figure 2.13-13) is called to compute F_S. Finally, LAPROP uses equations (2.13-1)) and (2.13-2)) or algorithm (2.13-8) to calculate F.

Block 10: If there are effects due to both multipath and diffraction, LAPROP calls MLTPTH (figure 2.13-14) to determine F_M, the propagation effects factor for multipath.

Blocks 11 and 12: If there are propagation effects due to both multipath and diffraction, then LAPROP uses logic analogous to that in Blocks 7 and 8 above to determine F_D.

Block 13: If there are propagation effects due to both multipath and diffraction, LAPROP uses equation (2.13-1)) to compute F.

Blocks 14 and 15: If all propagation effects are due to multipath, then LAPROP calls MLTPTH to determine F_M and sets $F = F_M$.

Block 16: LAPROP returns F^4 to the calling routine (TARGPU or TARGPD) where it is multiplied by the direct target signal received at the radar antenna. As indicated in Design Element 13-6, this is the factor for target returns. Design Element 13-6 indicates F^4 is not used for jammer returns since the jammer signal travels only from the jammer to the antenna, not a round trip from the antenna to the target back to the antenna. Thus, subroutine JAMMER computes the square root of F^4 to obtain F^2, the one-way pattern propagation factor.

Logical Flow for Knife Edge Diffraction

Figure 2.3-12 shows the logical flow for the calculation of F_K, the knife edge diffraction factor. The main routine for this calculation is KEDIFF. The numbered blocks are described below.

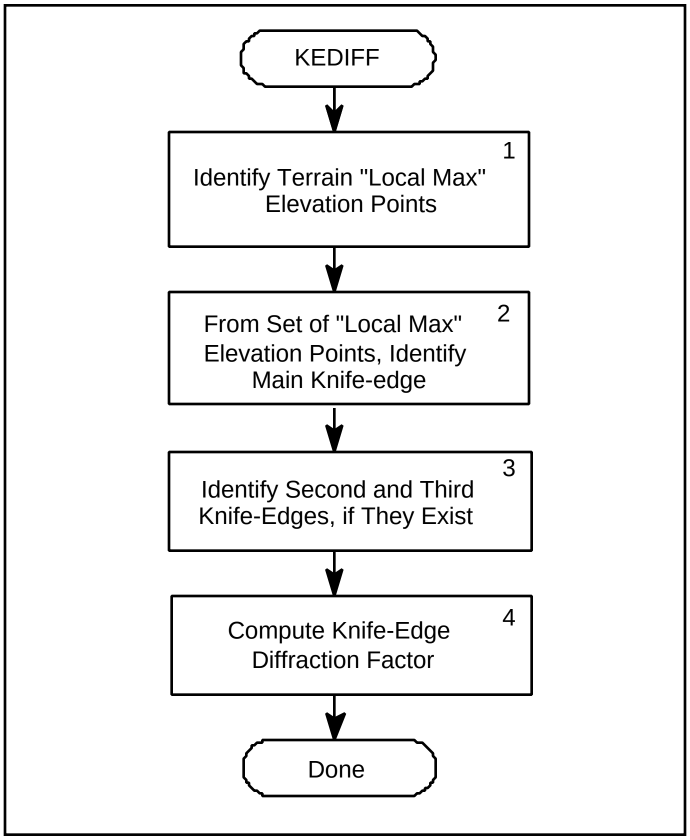

Figure 2.13-12 Knife Edge Diffraction Logical Flow

Block 1: Starting with i = 2, subroutine KEDIFF loops through all points in the terrain profile between the radar and the target. It searches for all local maxima; i.e, all points satisfying equation (2.13-29)). If no such points are found, F_K is set to 1.

Block 2: The set of all local maximum points $\{i_1, i_2, \ldots, i_N\}$ is ordered by increasing value of Δ_i/Δ_{i0} as defined in equations (2.13-3)) and (2.13-4)). Thus i_1 is the local maximum with minimum Δ_i/Δ_{i0}, and it is defined to be the main knife edge diffraction point M.

Block 3: KEDIFF loops through the set of remaining local maxima in the order described in Block 2. The first such point i_j between the radar and the M that is separated from M by at least ten other terrain points is defined to be the left knife edge diffraction point L. Similarly, the first such point between M and the target, which is separated from M by at least 10 other terrain points, is defined to be the right knife edge diffraction point R.

Block 4: KEDIFF calls subroutine DEYGOU to calculate F_K using equations (2.13-36)) through (2.13-47)).

Logical Flow for Spherical Earth Diffraction

Figure 2.13-13 shows the logical flow for the calculation of F_S, the spherical earth diffraction factor. The main routine for this calculation is SEDIFF. The numbered blocks are described below.

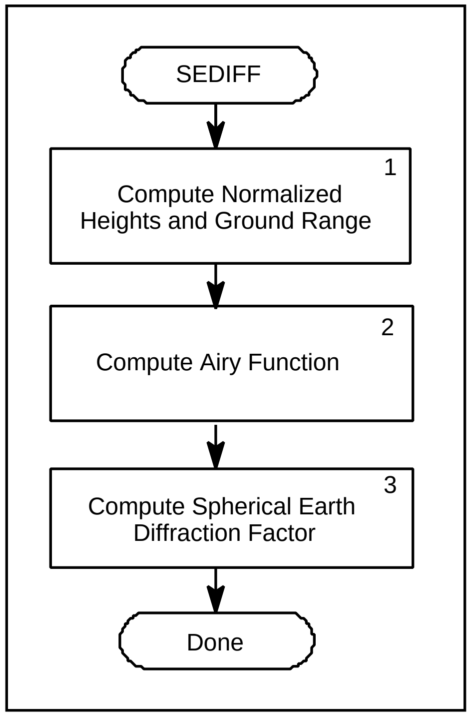

Figure 2.13-13 Spherical Earth Diffraction Logical Flow

Block 1: Subroutine SEDIFF begins by calling subroutine PARFIT to find the best parabolic fit to the terrain profile between the radar and the target. Based on this parabola (equation (2.13-52))) , SEDIFF uses equations (2.13-53)) through (2.13-56)) to find the normalized heights of radar and target and the normalized ground range from radar to target.

Block 2: In order to calculate the value of Fock's series (equation (2.13-49))), SEDIFF calls the function AIRY to compute $\overline{\mathrm{Ai}}$ as described in equations (2.13-61)) - (2.13-70)). Airy calls CONECT to implement equation (2.13-64)), GAUSSQ to implement equation (2.13-65)), and POWERS to implement equation (2.13-66)).

Block 3: SEDIFF evaluates the terms of Fock's series (equation (2.13-49))) and determines its convergence as described in the paragraphs following equation (2.13-51)). If Fock's series converges, SEDIFF calls OFFBOR and TGAIN to compute σ , and uses it to calculate F_S from equation (2.13-33)). If Fock's series diverges, then spherical diffraction is called divergent and is not used. Even if the series converges, values which produce $F_S \geq 2$ are physically impossible and hence are treated as if the series diverged. The maximum factor is two, which occurs when the phase difference is zero.

Logical Flow for Multipath

Figure 2.13-14 shows the logical flow for the calculation of F_M, the multipath effects factor. The main routine for this calculation is MLTPTH. The numbered blocks are discussed below.

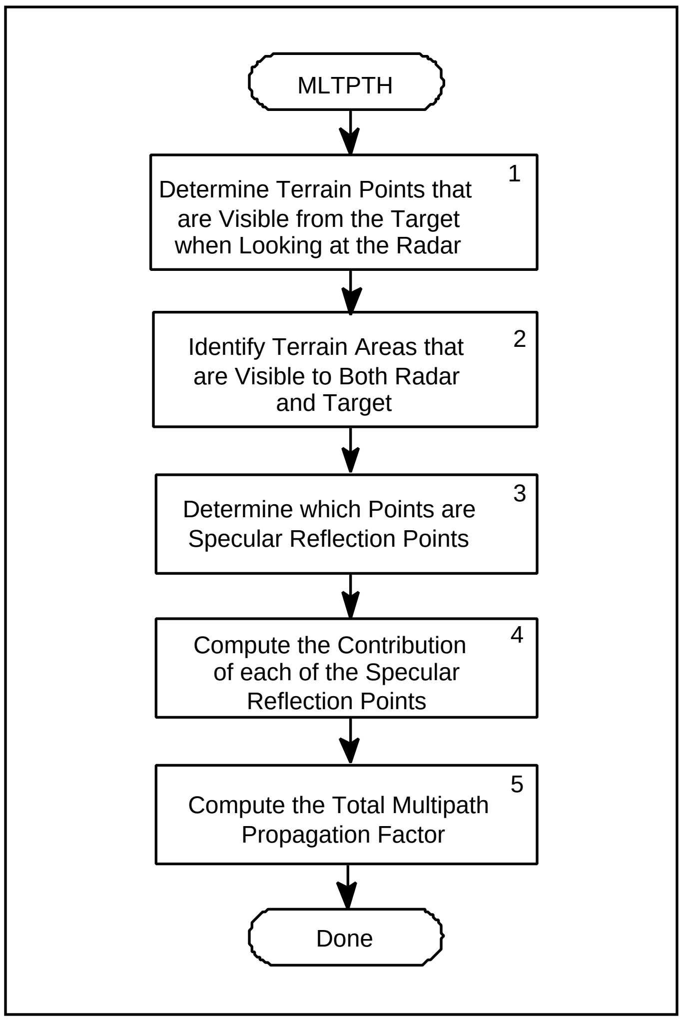

Figure 2.13-14 Multipath Logical Flow

Block 1: If the terrain roughness factor ρ_s is greater than zero, then subroutine MLTPTH loops through all points in the terrain profile between the target and the radar to determine which are visible to the target. This information is combined with previously obtained flags indicating which points are visible to the radar. This process yields a set of terrain points visible to both target and radar. These will be called visible terrain points and examined to determine whether or not they are specular reflection points. If $\rho_s \leq 0$, then MLTPTH skips to Block 5.

Block 2: MLTPTH groups each set of consecutive visible terrain points into visible areas. Each visible area will be examined for specular reflection points.

Block 3: To search for specular reflection points, MLTPTH loops on visible areas and then loops on terrain points within the visible area. Each point i is examined as described in equations (2.13-25)) and (2.13-26)) to determine whether or not it is a specular point. If so, then the grazing angle is determined using equation (2.13-27)).

Block 4: If any specular points are found in a visible area, each one is checked to see if the returned pulse along the direct path overlaps the returned pulse along the reflected path. If not, then the energy returned along the reflected path will not affect the direct path signal, so this point will not contribute to a multipath effects factor, and it is not considered to be a specular point. If the pulses do overlap, then the width of the first Fresnel zone at the point is calculated using equation (2.13-24)). The specular point with maximum Fresnel zone width in each area is selected as the only point contributing to the multipath effect in that area (the Fresnel zone is assumed to encompass any other effects in the area.) Subroutine RFLECT is called to find the reflection coefficient ρ_i using equations (2.13-19)) and (2.13-20)). If the "terrain" is water (LCOVER=7), then equation (2.13-21)) is used to compute a terrain roughness coefficient. OFFBOR and TGAIN are called to find the appropriate gain G_B of equation (2.13-18)), so that MLTPTH can compute the i^{th} term of the summation in that equation.

Block 5: MLTPTH calls OFFBOR and TGAIN to calculate G_D, the gain in the direction of the target. If at least one specular point was found in the terrain profile, then equation (2.13-18)) is used to calculate F_M. Otherwise, if any visible areas were found, equations (2.13-25)) and (2.13-26)) are used to examine the terrain patch between the radar and the first point in the terrain profile. If a geometric specular point is found there and if its returned reflected pulse overlaps with the returned direct pulse, then the first terrain point is designated as a specular point and the reflection coefficient and the terrain roughness coefficient are calculated as in Block 4. Equation (2.13-18)) is then used to calculate F_M. (Note that there is no need to calculate the width of the first Fresnel zone for only one specular point, since the single w_i term can be canceled from numerator and denominator.) If no specular points are found in the entire terrain profile (including the case when none are looked for because $\rho_s \leq 0$), then equation (2.13-18)) collapses so that F_M equals the square root of G_D.

Multipath and Diffraction Inputs and Outputs

The only output of the Multipath and Diffraction functional element is the pattern propagation factor listed in table 2.13-2.

Table 2.13-2 Multipath/Diffraction Outputs

VARIABLE NAME	DESCRIPTION
FTO4TH	Pattern propagation factor (including antenna gain) to be included in calculations of signals received. The square root of this value is used as the pattern propagation factor for jammers.

User inputs which affect multipath and diffraction are given in table 2.13-3. In addition to the specific input variables listed in the table, this FE also uses values derived from user-input radar site coordinates and terrain data.

Table 2.13-3 User Inputs for Multipath/Diffraction

DATABLOCK NAME	VARIABLE NAME	DESCRIPTION
DATARADR	PULWID	Radar pulse width (μsec)
DATARADR	IPOLAR	Antenna polarization: 0 = vertical, 1 = horizontal
DATARADR	ZTENNA	Antenna height (meters)
DATARADR	FREQIN	Radar frequency (MHz), used to calculate wavelength (RLAMDA)
DATAJAMR	CFREQJ	Center frequency of a jammer (MHz)
DATASEKE	IPROP	0 = radar antenna gain is only propagation factor 1 = propagation factor includes antenna gain plus refraction, diffraction, and multipath effects
DATASEKE	EPSLN1	Relative dielectric constant of the terrain
DATASEKE	SIGMHO	Terrain conductivity (mho/m)
DATASEKE	RROUGH	Terrain roughness factor/scattering coefficient for land. Not used for water.
DATASEKE	RKFACT	Refractivity factor (commonly called "four-thirds earth")
DATASEKE	TAURLX	Relaxation constant, for use when LCOVER = 7 (water) only. Used to determine terrain roughness factor for pattern propagation factor over the sea.
DATASEKE	WNDKNO	Wind speed, for use when LCOVER = 7 (water) only. Used to determine terrain roughness factor for pattern propagation factor over the sea.
DATAREFL	LCOVER	Lincoln Labs clutter reflectivity data land cover indicator. Used in this FE only to check for LCOVER = 7 (water), because different formulas for the terrain roughness factor are used over land and over sea.

The flag IPROP determines whether or not multipath and diffraction will be evaluated. EPSLN1 is the ratio of the dielectric constant for the terrain to the dielectric constant for free space; typical values range from 3 for dry soil to 80 for fresh water. SIGMHO (σ) is typically 10^{-3} for land and

4.0 for sea. RROUGH (ρ_s) is an empirical correction factor that is wavelength dependent; some values from tests conducted by Lincoln Laboratory range from 0.2 to 1.0. RKFACT (K) accounts for atmospheric bending of radar waves; typically set at 4/3, but values can vary widely.

Inputs and outputs for the principal subroutines which directly implement the multipath/ diffraction functional element are given in tables 2.13-4 through 2.13-19.

Table 2.13-4 Function Airy Inputs and Outputs

FUNCTION: AIRY					
INPUTS			**OUTPUTS**		
NAME	**TYPE**	**DESCRIPTION**	**NAME**	**TYPE**	**DESCRIPTION**
ZARGMT	Argument	Complex argument of Airy function	AIRY	Function value returned	Airy function evaluated at ZARGMT

Table 2.13-5 Function CONECT Inputs and Outputs

FUNCTION: CONECT					
INPUTS			**OUTPUTS**		
NAME	**TYPE**	**DESCRIPTION**	**NAME**	**TYPE**	**DESCRIPTION**
ZARGMT	Argument	Complex argument of Airy function	CONECT	Function value returned	Value of Airy function in region 1, the connection region of the complex plane

Table 2.13-6 Subroutine DEYGOU Inputs and Outputs

SUBROUTINE: DEYGOU					
INPUTS			**OUTPUTS**		
NAME	**TYPE**	**DESCRIPTION**	**NAME**	**TYPE**	**DESCRIPTION**
ILEFT	Argument	Index of secondary knife edge diffraction point between radar and main point	FSUBK	Argument	Knife edge diffraction effects factor
IRIGHT	Argument	Index of secondary knife edge diffraction point between main point and target			
IMAIN	Argument	Index of main knife edge diffraction point			
DRATIO	Common PRFILE	Array of clearance ratios for terrain profile points			

Table 2.13-6 Subroutine DEYGOU Inputs and Outputs

SUBROUTINE: DEYGOU					
INPUTS			**OUTPUTS**		
NAME	**TYPE**	**DESCRIPTION**	**NAME**	**TYPE**	**DESCRIPTION**
INDXFC	Common PRFILE	Index to minimum DRATIO			
XPROFL	Common PRFILE	Array of x coordinates of profile points			
ZPROFL	Common PRFILE	Array of z coordinates of profile points			
XTPROF	Common GEOMRT	x coordinate of target			
ZTPROF	Common GEOMRT	z coordinate of target			
RLAMDA	Common RADPAR	Wavelength (meters)			
EPSLNT	Common GEOMRT	Elevation angle of target with respect to radar			
EPSLNR	Common GEOMRT	Elevation pointing angle of radar boresight			
ALPHAR	Common GEOMRT	Azimuth pointing angle of radar boresight			
ALPHAT	Common GEOMRT	Azimuth angle of target with respect to radar			
DSQRT2	Common CONSTR	Square root of 2			

Table 2.13-7 Subroutine FIRST Inputs and Outputs

SUBROUTINE: FIRST					
INPUTS			**OUTPUTS**		
NAME	**TYPE**	**DESCRIPTION**	**NAME**	**TYPE**	**DESCRIPTION**
HAMMSL	Common SITCOM	Radar antenna height	FRCMIN	Argument	Minimum value of DRATIO
NPROFL	Common PRFILE	Number of points in terrain profile	DRATIO	Common PRFILE	Array of clearance ratios for terrain profile
RLAMDA	Common RADPAR	Wavelength (meters)	INDXFC	Common PRFILE	Index to minimum value of DRATIO
TANEPT	Common GEOMRT	Tangent of target elevation angle			
XPROFL	Common PRFILE	Array of x coordinates of terrain points			
ZPROFL	Common PRFILE	Array of z coordinates of terrain points			

Table 2.13-7 Subroutine FIRST Inputs and Outputs

SUBROUTINE: FIRST					
INPUTS			OUTPUTS		
NAME	TYPE	DESCRIPTION	NAME	TYPE	DESCRIPTION
XTPROF	Common GEOMRT	x coordinate of target			
ZTPROF	Common GEOMRT	z coordinate of target			

Table 2.13-8 Subroutine FRESNEL Inputs and Outputs

SUBROUTINE: FRESNEL					
INPUTS			OUTPUTS		
NAME	TYPE	DESCRIPTION	NAME	TYPE	DESCRIPTION
XARGMT	Argument	Real argument of Fresnel sine and cosine functions	COSINT	Argument	Fresnel cosine integral evaluated at XARGMT
HALFPI	Common CONSTR	$\pi/2$	SININT	Argument	Fresnel sine integral evaluated at XARGMT
SR2PO4	Common CONSTR	$\frac{\sqrt{2\pi}}{4}$			

Table 2.13-9 Function GAUSSQ Inputs and Outputs

FUNCTION: GAUSSQ					
INPUTS			OUTPUTS		
NAME	TYPE	DESCRIPTION	NAME	TYPE	DESCRIPTION
ZARGMT	Argument	Complex argument of Airy function	GAUSSQ	Function value returned	Value of Airy function in region 2, the integral region of the complex plane

Table 2.13-10 Subroutine KEDIFF Inputs and Outputs

SUBROUTINE: KEDIFF					
INPUTS			OUTPUTS		
NAME	TYPE	DESCRIPTION	NAME	TYPE	DESCRIPTION
DRATIO	Common PRFILE	Array of clearance ratios for terrain profile	FSUBK	Argument	Knife edge diffraction effects factor
NPROFL	Common PRFILE	Number of points in terrain profile			
ZPROFL	Common PRFILE	Array of z coordinates of terrain points			

Table 2.13-11 Subroutine LAPROP Inputs and Outputs

SUBROUTINE: LAPROP					
INPUTS			OUTPUTS		
NAME	TYPE	DESCRIPTION	NAME	TYPE	DESCRIPTION
ALPHAR	Common GEOMRT	Azimuth coordinate of radar	FTO4TH	Argument	Total propagation effects factor
EPSLNR	Common GEOMRT	Elevation coordinate of radar			
ALPHAT	Common GEOMRT	Azimuth coordinate of target			
EPSLNT	Common GEOMRT	Elevation coordinate of target			
IFLSAM	Common RADPAR	Flag. Equals 1 if and only if receiver gain = transmitter gain, otherwise equals 0			
LPPROP	Common LOGICL	Conversion of user input IPROP from 1/0 to TRUE/ FALSE			

Table 2.13-12 Subroutine LINFIT Inputs and Outputs

SUBROUTINE: LINFIT					
INPUTS			OUTPUTS		
NAME	TYPE	DESCRIPTION	NAME	TYPE	DESCRIPTION
DELTAG	Common PRFILE	Ground range increment between terrain points	ALINE0	Argument	a_0 in linear equation $y = a_0 + a_1 x$
NPROFL	Common PRFILE	Number of points in terrain profile	ALINE1	Argument	a_1 in linear equation $y = a_0 + a_1 x$
ELVLMSL	Common PRFILE	Array of heights (MSL) of terrain points			

Table 2.13-13 Subroutine MLTPTH Inputs and Outputs

SUBROUTINE: MLTPTH					
INPUTS			**OUTPUTS**		
NAME	**TYPE**	**DESCRIPTION**	**NAME**	**TYPE**	**DESCRIPTION**
NPROFL	Common PRFILE	Number of points in terrain profile	FSUBM	Argument	Multipath effects factor
VISIBL	Common LOGICL	Array of logical values for profile points. True if the point is visible (not masked) from the radar. Note this definition changes in this subroutine	VISIBL	Common LOGICL	Array of logical values for profile points. True if the point is visible from both the radar and the target. Note this definition has changed during this subroutine
TANEPP	Common PRFILE	Array of tangents of elevation angles of terrain points			
XPROFL	Common PRFILE	Array of x coordinates of terrain points			
ZPROFL	Common PRFILE	Array of z coordinates of terrain points			
ALPHAR	Common GEOMRT	Azimuth pointing angle of radar boresight			
EPSLNR	Common GEOMRT	Elevation pointing angle of radar boresite			
ALPHAT	Common GEOMRT	Azimuth coordinate of target wrt radar			
EPSLNT	Common GEOMRT	Elevation coordinate of target wrt radar			
PULWID	Common RADPAR	Pulse width (seconds)			
VLIGHT	Common CONSTR	Velocity of light (m/sec)			
RROUGH	Common ENVIRO	Terrain roughness factor / scattering coefficient			
XTPROF	Common GEOMRT	x-coordinate of target in profile coordinate system			
ZTPROF	Common GEOMRT	z-coordinate of target in profile coordinate system			
HAMMSL	Common SITCOM	Height of radar antenna above mean sea level (meters)			
RANGET	Common GEOMRT	Slant range from radar to target			
LCOVER	Common RFLCOM	Type of landcover; 7 = water			
RLAMDA	Common RADPAR	Radar wavelength (meters)			

Table 2.13-14 Subroutine PARFIT Inputs and Outputs

SUBROUTINE: PARFIT					
INPUTS			OUTPUTS		
NAME	TYPE	DESCRIPTION	NAME	TYPE	DESCRIPTION
DELTAG	Common PRFILE	Ground range increment between terrain points	APARA0	Argument	a_0 in parabolic equation $y = a_0 + a_1x + a_2x^2$
NPROFL	Common PRFILE	Number of points in terrain profile	APARA1	Argument	a_1 in parabolic equation $y = a_0 + a_1x + a_2x^2$
ELVLMSL	Common PRFILE	Array of heights (MSL) of terrain points	APARA2	Argument	a_2 in parabolic equation $y = a_0 + a_1x + a_2x^2$

Table 2.13-15 Function POWERS Inputs and Outputs

SUBROUTIND: POWERS					
INPUTS			OUTPUTS		
NAME	TYPE	DESCRIPTION	NAME	TYPE	DESCRIPTION
ZARGMT	Argument	Complex argument of Airy function	POWERS	Function value returned	Value of Airy function in region 3, the power series region of the complex plane

Table 2.13-16 Subroutine PROFIL Inputs and Outputs

SUBROUTINE: PROFIL					
INPUTS			OUTPUTS		
NAME	TYPE	DESCRIPTION	NAME	TYPE	DESCRIPTION
DELTAG	Common PRFILE	Ground range between terrain points	MASKED	Common LOGICL	Flag to indicate target masked (T) or unmasked (F)
SIALPT COALPT	Common GEOMRT	Sine and cosine of the target azimuth angle	NPROFL	Common PRFILE	Number of points in terrain profile between radar and target
GRANGT	Common GEOMRT	Ground range from radar to target			
TANEPT	Common GEOMRT	Tangent of the elevation angle of the target			

Table 2.13-17 Subroutine RFLECT Inputs and Outputs

SUBROUTINE: RFLECT					
INPUTS			OUTPUTS		
NAME	TYPE	DESCRIPTION	NAME	TYPE	DESCRIPTION
GAMMA	Argument	Grazing angle	RCOEFF	Argument	Reflection coefficient
IPOLAR	Common RADPAR	Polarization flag 0 = vertical 1 = horizontal			
YSQUAR	Common COMPLX	Square of the normalized admittance			

Table 2.13-18 Subroutine SECOND Inputs and Outputs

SUBROUTINE: SECOND					
INPUTS			OUTPUTS		
NAME	TYPE	DESCRIPTION	NAME	TYPE	DESCRIPTION
DELTAG	Common PRFILE	Ground range between points in terrain	HMDZRO	Argument	Ratio of terrain height to Fresnel clearance at point INDXFC.
ELVMSL	Common PRFILE	Array of heights (MSL) of terrain points (meters)			
INDXFC	Common PRFILE	Index of minimum value of DRATIO			
GRANGT	Common GEOMRT	Ground range from radar to target			
RLAMDA	Common RADPAR	Wavelength (meters)			

Table 2.13-19 Subroutine SEDIFF Inputs and Outputs

SUBROUTINE: SEDIFF					
INPUTS			OUTPUTS		
NAME	TYPE	DESCRIPTION	NAME	TYPE	DESCRIPTION
DRATIO	Common PRFILE	Array of clearance ratios for terrain points	FSUBS	Argument	Spherical earth diffraction effects factor
INDXFC	Common PRFILE	Index to minimum value in DRATIO	CONVRG	Argument	Flag indicating whether spherical earth diffraction converges (T) or not (F)
XPROFL	Common PRFILE	Array of x coordinates of terrain points			
ZPROFL	Common PRFILE	Array of z coordinates of terrain points			

Table 2.13-20 Subroutine SEKINT Inputs and Outputs

SUBROUTINE: SEKINT					
INPUTS			OUTPUTS		
NAME	TYPE	DESCRIPTION	NAME	TYPE	DESCRIPTION
IPPROP	Common ENVIRO	Input flag 0 = do not calculate propagation factor 1 = calculate propagation factor	LPPROP	Common LOGICL	Conversion of IPPROP 0 = FALSE 1 = TRUE
REZERO	Common ENVIRO	Radius of earth at mean sea level (meters)	DELTAG	Common PRFILE	Distance between points in terrain profile (meters)
RKFACT	Common ENVIRO	Refractivity factor to be multiplied times radius of earth (commonly 4/3)	REARTH	Common ENVIRO	Radius of earth converted to account for refractivity (meters)
FREQIN	Common RADPAR	Radar frequency (MHz)	YSQUAR	Common COMPLX	Factor for use in calculating reflection coefficient for smooth plane
TAURLX	Common ENVIRO	Relaxation constant for use when LCOVER = 7	SINBET COSBET	Common TRIGON	Arrays of sines and cosines of the angles $ß_i$ between position vectors to radar and to point i * DELTAG distant from radar, assuming origin at center of earth
EPSLN1	Common ENVIRO	Relative dielectric constant of the terrain	SBETAP CBETAP	Common TRIGON	Same as SINBET and COSBET except with radius of earth adjusted for refraction factor
SIGMHO	Common ENVIRO	Terrain conductivity (mho/m)	RROUGH	Common ENVIRO	Output only for LCOVER=7. Factor used in calculation of reflection coefficient when "terrain" is water
WNDKNO	Common ENVIRO	Wind speed (knots)			
RLAMDA	Common RADPAR	Wavelength (meters)			
LCOVER	Common RFLCOM	Type of land cover; 7 = water			
PI TWOPI	Common CONSTR	π 2π			

Table 2.13-21 Subroutine VISIBLE Inputs and Outputs

SUBROUTINE: VISIBLE					
INPUTS			**OUTPUTS**		
NAME	**TYPE**	**DESCRIPTION**	**NAME**	**TYPE**	**DESCRIPTION**
COAZIN	Argument	Cosine of azimuth angle of input point (target position)	ELVMSL	Common PRFILE	Array of heights (MSL) of the terrain points between the radar and the input point (meters)
IPROFL	Argument	Number of points in the terrain profile between the radar and the input point	TANEPP	Common PRFILE	Array of tangents of elevation angles of the terrain points
SIAZIN	Argument	Sine of the azimuth angle of the input point	TANMAX	Common PRFILE	Maximum value in array TANEPP
SIPHIS COPHIS	Common SITCOM	Sine and cosine of radar site latitude	VISIBL	Common LOGICL	Array of logical values for profile points. True if the point is visible (not masked) from the radar. This definition will change in subroutine MLTPTH.
SINBET COSBET	Common TRIGON	Arrays of sines and cosines of the angles $ß_i$ between position vectors to radar and to point i * DELTAG distant from radar, assuming origin at center of the earth	XPROFL	Common PRFILE	Array of x coordinates of terrain profile points
SBETAP CBETAP	Common TRIGON	Same as SINBET and COSBET except with radius of earth adjusted for refraction factor	ZPROFL	Common PRFILE	Array of z coordinates of terrain profile points
SITLAM	Common SITCOM	Radar site longitude (radius)			
REARTH	Common ENVIR	Radius of earth converted to account for refractivity (meters)			
HAMMSL	Common SITCOM	Height of radar antenna above mean sea level (meters)			
PI TWOPI HALFPI	Common CONSTR	π 2π $\pi/2$			

2.13.4 Assumptions and Limitations

Only single bounce (in each direction) specular reflection paths are used for calculating multipath. Neither multiple bounce paths or diffuse reflection are considered.

The pattern propagation factor generated for the path from the radar to a randomly selected point (i.e., the target or a jammer) is identical to the propagation factor for the path from that point to the radar.

Positional data used to access the DMA terrain data base are calculated with integer arithmetic, accurate to the nearest second of latitude/longitude (about 30 meters). DMA DTED terrain data are available in a grid, with points separated by about 90 meters in ground range.

The propagation factor is calculated based only on terrain points that lie in the vertical plane defined by the radar, the target, and the center of the earth. Propagation effects due to points outside this plane are not considered.

The mean earth radius at sea level is assumed to be a constant, 6,371,007 meters.

All terrain within the volume viewed by the radar is assumed to have the same terrain roughness factor, terrain dielectric constant, ground conductivity factor, and refractivity factor.